Advanced Discrete Structure Homework 3 Tutorial

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Question 1

Solve the following recurrence relations:

(a) $a_n = 3a_n - 1 + 4a_{n-2}$, $a_o = a_1 = 1$

(b) $a_n = 2a_{n-1} + (-1)^n$, $a_0 = 2$

(c) $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$,

$a_0 = a_1 = 1$,$a_2 = 2$

Question 1

You can use the ways to told in class:

1. Find out the particular solutions and the homogeneous solutions.
2. Use generating functions.

Question 2

Solve the following recurrence relations when $a_0 = 1$:

(a) $a_n^2 = 2a_{n-1}^2 + 1$ (*Hint*: Let $b_n = a_n^2$)

(b) $a_n = -na_{n-1} + n!$

(*Hint*: Define an appropriate b_n as in part (a).)

Question 3

Find functional equations for the generating functions whose coefficients satisfy the following relations:

(a) $a_n = a_{n-1} + n(n-1)$, $a_0 = 1$

(b) $a_n = \sum_{i=2}^{n-2} a_i a_{n-i} (n \geq 3)$,

$$a_0 = a_1 = a_2 = 1$$

Question 4

Find and solve a recurrence relation for the number of n-digit ternary sequences in which no 1 appears to the right of any 2.

(ternary: composed of three items)

Hint

Such sequences ends with 0:

$$a_n = a_{n-1} + b_{n-1} + c_{n-1}$$

Such sequences ends with 1:

$$b_n = a_{n-1} + b_{n-1}$$

Such sequences ends with 2:

$$c_n = a_{n-1} + b_{n-1} + c_{n-1}$$

All Such sequences:

$$d_n = a_n + b_n + c_n$$

Hint

Simplify them

and solve them by generating functions.

Question 5

How many ways are there to completely cover a $2 \times n$ rectangle with 2×1 dominos?

Question 5

For n = 3:

Question 6 (Challenge)

How many ways are there to completely cover a $3 \times n$ rectangle with 2×1 dominos?

Question 6 (Challenge)

For n = 4: