

CS5319 ADVANCED DISCRETE STRUCTURE

Homework 3

Due: 3:20 pm, November 3, 2011 (before class)

1. Solve the following recurrence relations:

(a) $a_n = 3a_{n-1} + 4a_{n-2}$, $a_0 = a_1 = 1$

(b) $a_n = 2a_{n-1} + (-1)^n$, $a_0 = 2$

(c) $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$, $a_0 = a_1 = 1, a_2 = 2$

2. Solve the following recurrence relations when $a_0 = 1$:

(a) $a_n^2 = 2a_{n-1}^2 + 1$ (*Hint: Let $b_n = a_n^2$*)

(b) $a_n = -na_{n-1} + n!$ (*Hint: Define an appropriate b_n as in part (a).*)

3. Find the generating functions whose coefficients satisfy the following relations:

(a) $a_n = a_{n-1} + n(n-1)$, $a_0 = 1$

(b)

$$a_n = \sum_{i=2}^{n-2} a_i a_{n-i} \text{ for } n \geq 3, \text{ and } a_0 = a_1 = a_2 = 1.$$

4. Find and solve a recurrence relation for the number of n -digit ternary sequences in which no 1 appears to the right of any 2.

5. How many ways are there to completely cover a $2 \times n$ rectangle with 2×1 dominoes?

6. (Challenging: No marks) How many ways are there to completely cover a $3 \times n$ rectangle with 2×1 dominoes?