CS5319 Advanced Discrete Structure

Homework 2

Due: 3:20 pm, October 18, 2011 (before class)

1. Use binomial expansions or combinatorial arguments to evaluate the following sums:

   (a)
$$\sum_{k=0}^{m} \binom{m}{k} \binom{n}{r+k}$$

   (b)
$$\sum_{k=0}^{r} (-1)^k \binom{n}{k} \binom{n}{r-k}$$

   (c)
$$\sum_{k=0}^{n} 2^k \binom{n}{k}$$

2. Find the coefficient of x^{12} in
$$\frac{x+3}{1-2x+x^2}.$$

3. (a) Find the ordinary generating function of the sequence $(a_0, a_1, a_2, ...)$ where a_r is the number of ways in which the sum r will show when two distinct dice are rolled, with the first one showing even and the second one showing odd.

   (b) Find the ordinary generating function of the sequence $(a_0, a_1, a_2, ...)$ where a_r is the number of ways in which the sum r will show when 10 distinct dice are rolled, with five of them showing even and the other five showing odd.

4. How many ways are there to collect \$24 from 4 children and 6 adults if each person gives at least \$1, but each child can give at most \$4 and each adult at most \$7?

5. Find the exponential generating function with:

   (a) $a_r = 1/(r+1)$

   (b) $a_r = r!$

6. Find the number of n-digit strings generated from the alphabet $\{0, 1, 2, 3, 4\}$ whose *total number* of 0's and 1's is even.

7. (Challenging: No marks) Show that the number of partitions of n is equal to the number of partitions of $2n$ into n parts. (Hint: Use Ferrers graph.)

8. (Challenging: No marks) Show that for any $s > 1$,
$$\sum_{n=1}^{\infty} \frac{1}{n^s} \equiv \prod_{p \text{ prime}} \frac{1}{1-p^{-s}}.$$

   The sum of the left-hand side is popularly known as the Riemann zeta function, $\zeta(s)$, and the overall identity is known as the Euler product formula.