Advanced Discrete Structure Homework 1 Tutorial

Simon Chang

Let’s start with

HOMEWORK QUESTIONS

Question 1

Suppose a coin is tossed 12 times and there are 3 heads and 9 tails.

How many such sequences are there in which there are at least 5 tails in a row?

Ex: Use 1 for a head and 0 for a tail.

100000000110

Is It Right?

Concatenate five 0s together and regard it as a big object: **00000**

Ex:

100**00000**0110

Is It Right?

Therefore, the answer is

$$\frac{8!}{3!\,4!\,1!}$$

Is It Right?

WRONG!!

These Are Considered The Same:

1**00000**000110
10**00000**00110
100**00000**0110
1000**00000**110

01**00000**01010
010**00000**1010

01**00000**00110
010**00000**0110
0100**00000**110

11**00000**00001
110**00000**0001
1100**00000**001
11000**00000**01
110000**00000**1

Hint

There are $2n + 1$ seats in a congress, to be divided among three parties. In how many ways will some party obtain a majority of the seats ?

Question 2

How many non-negative integer solutions are there to the equation

$$2x_1 + 2x_2 + x_3 + x_4 = 12?$$

Ex:

$$(x_1, x_2, x_3, x_4) = (1,2,3,3)$$

Hint

The easiest way is to solve it case by case.

Question 3 (a)

Show that the total number of permutations of p red balls and 0, or 1, or 2, …, or q white balls is

$$\frac{p!}{p!} + \frac{(p+1)!}{p!\,1!} + \frac{(p+2)!}{p!\,2!} + \cdots + \frac{(p+q)!}{p!\,q!}$$

Question 3 (a)

Just explain it by words.

Very easy.

Question 3 (b)

Show that the sum in part (a) is

$$\frac{(p+q+1)!}{(p+1)!\,q!}$$

Hint

What does $\frac{(p+q+1)!}{(p+1)!q!}$ means?

How to transform it into the permutations of balls in part (a)?

Question 3 (c)

Show that the total number of permutations of 0, or 1, or 2, …, or *p* red balls with 0, or 1, or 2, …, or *q* white balls is

$$\frac{(p+q+2)!}{(p+1)!\,(q+1)!}-1$$

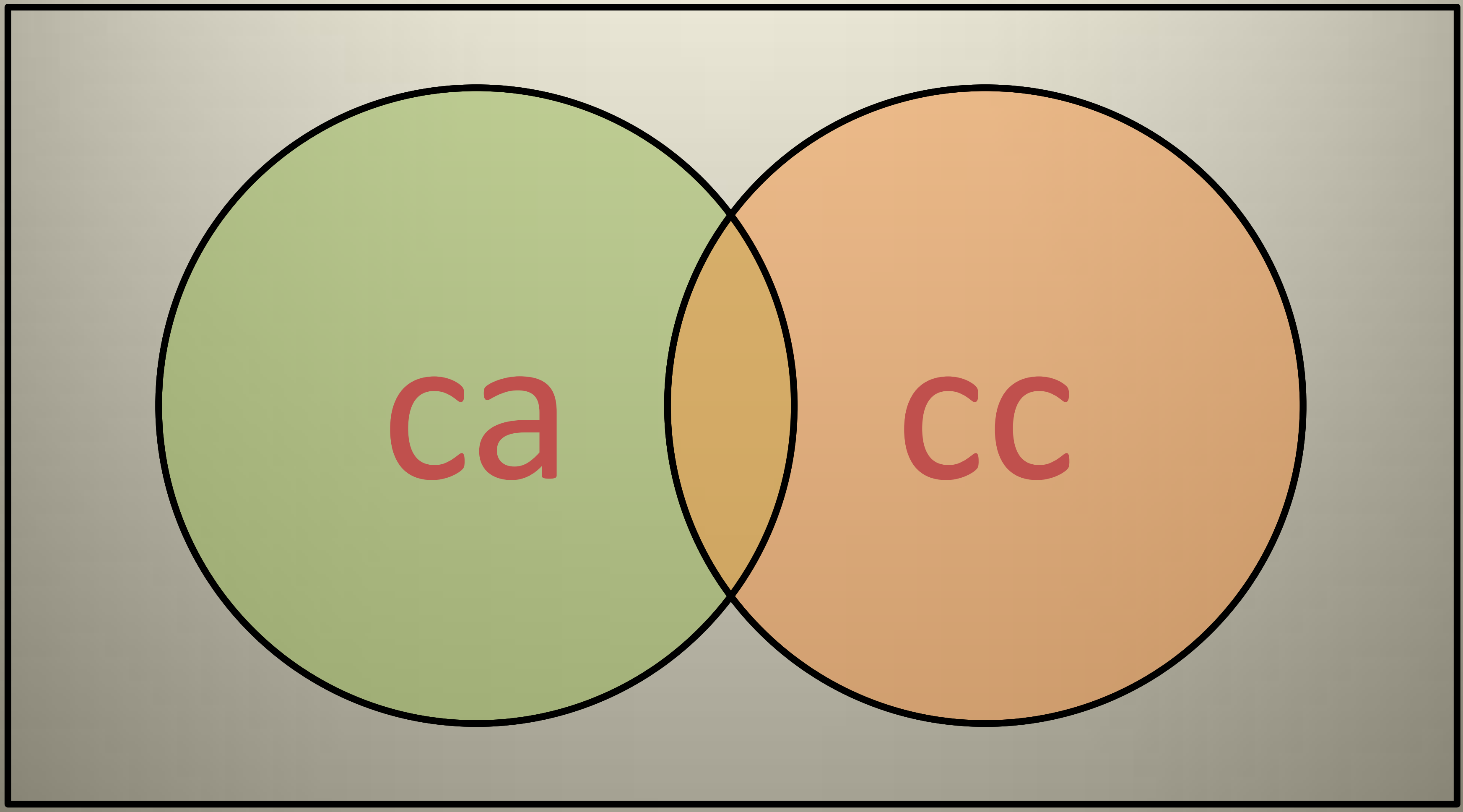
Question 3 (c)

You should be able to explain this if you solved part (b).

Question 4

How many arrangements are there of seven *a*s, eight *b*s, three *c*s, and six *d*s with no occurrence of the consecutive pairs *ca* or *cc*?

Hint



Question 5

How many ways are there to distribute 25 dffierent presents to four people (including the boss) at an office party so that the boss receives exactly twice as many presents as the second popular person?

Question 5

Ex:

12	3	6	4
Boss	A	B	C
		↑	

The second popular person

Question 6

Professor Grinch's telephone number is 6328363. Mickey remembers the collections of digits but not their order, except that he knows the first 6 is before the first 3. How many arrangements of these digits with this constraint are there?

Ex:

8 6 2 3 3 6 3

6 6 8 2 3 3 3

Hint

Ignore 6s and 3s at first.

Question 7(Challenging: No marks)

A man has seven friends. How many ways are there to invite a different subset of three of these friends for a dinner on seven successive nights such that each pair of friends are together at just one dinner?

Question 7(Challenging: No marks)

Ex: Suppose his friends are A, B, C, D, E, F, and G.

Day 1: A, B, C

Day 2: A, D, E

Day 3: A, F, G

Day 4: B, D, G

Day 5: B, E, F

Day 6: C, D, F

Day 7: C, E, G

Question 7(Challenging: No marks)

A wrong one:

Day 1: B, E, F

Day 2: A, D, E

Day 3: C, E, G

Day 4: B, D, G

Day 5: A, B, C

Day 6: C, D, E

Day 7: A, F, G

Question 7(Challenging: No marks)

A wrong one:

Day 1: B, E, F

Day 2: A, D, E

Day 3: C, E, G

Day 4: B, D

Day 5: A, B, C

Day 6: C, D, E

Day 7: A, F, G

We meet again!

Question 7(Challenging: No marks)

You have no grade if you don’t do this.

You have no grade if you do this, either.

Hope these gives you some ideas.

INTERESTING QUESTIONS

Question 1

From n distinct integers, two groups of integers are to be selected with x integers in the first group and y integers in the second group, where $x + y \leq n$. In how many ways can the selection be made such that the smallest integer in the first group is larger than the largest integer in the second group?

Question 1

The smallest integer in the first group is larger than the largest integer in the second group

$$\downarrow$$

Every integer in the first group is larger than everyone in the second group

Question 1

The quick way:

Pick x + y integers: C(n, x + y)

Pick x larger integers: 1

$$C(n, x + y) \times 1 = C(n, x + y)$$

Question 2

In how many ways can four distinct integers be selected from 1 to 300 such that their sum is a multiple of 3?

Question 3

There are 15 lines on the plane.

Suppose that four lines are parallel to each other, while any two of the others have an intersection. How many line segments are there?

Question 3

Number of intersections: $\binom{15}{2} - \binom{4}{2}$

Each intersection produce 2 additional segments.

Total number of segments: $15 + 2(\binom{15}{2} - \binom{4}{2})$