CS5319 Advanced Discrete Structure

Homework 1

Due: 3:20 pm, October 4, 2011 (before class)

1. Suppose a coin is tossed 12 times and there are 3 heads and 9 tails. How many such sequences are there in which there are at least 5 tails in a row?

2. How many non-negative integer solutions are there to the equation $2x_1+2x_2+x_3+x_4 = 12$?

3. (a) Show that the total number of permutations of p red balls and 0, or 1, or 2, ..., or q white balls is
$$\frac{p!}{p!} + \frac{(p+1)!}{p!1!} + \frac{(p+2)!}{p!2!} + ... + \frac{(p+q)!}{p!q!}.$$

   (b) Show that the sum in part (a) is
$$\frac{(p+q+1)!}{(p+1)!q!}.$$

   (c) Show that the total number of permutations of 0, or 1, or 2, ..., or p red balls with 0, or 1, or 2, ..., or q white balls is
$$\frac{(p+q+2)!}{(p+1)!(q+1)!} - 1.$$

4. How many arrangements are there of seven as, eight bs, three cs, and six ds with no occurrence of the consecutive pairs ca or cc?

5. How many ways are there to distribute 25 different presents to four people (one of them is the boss) at an office party so that the boss receives exactly twice as many presents as the second popular person?

6. Professor Grinch's telephone number is 6328363. Mickey remembers the collections of digits but not their order, except that he knows the first 6 is before the first 3. How many arrangements of these digits with this constraint are there?

7. (Challenging: No marks) A man has seven friends. How many ways are there to invite a different subset of three of these friends for a dinner on seven successive nights such that each pair of friends are together at just one dinner?