CS 5319
Advanced Discrete Structure

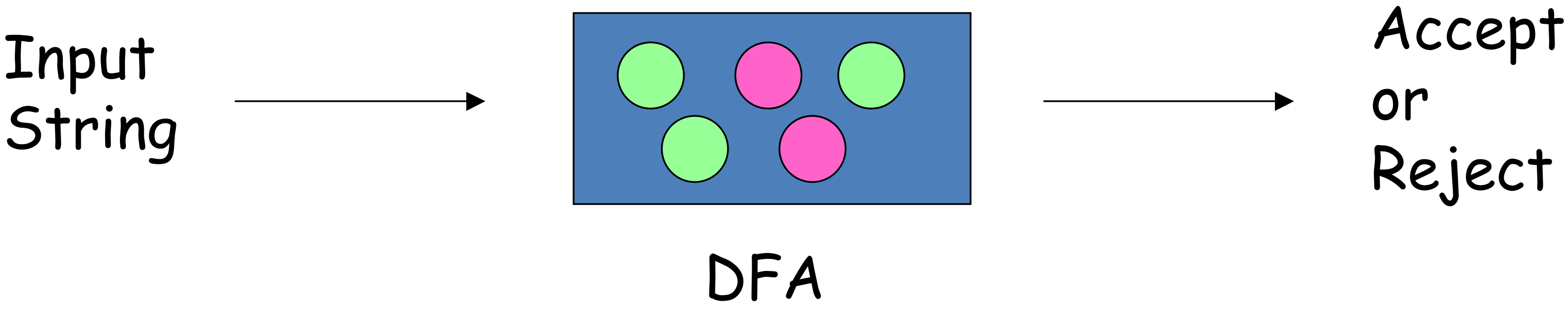
Lecture 15:
Introduction to Automata Theory

Outline

- Deterministic Finite Automaton (DFA)
- Pumping Lemma
- Non-Deterministic Finite Automaton (NFA)
- Equivalence between DFA and NFA

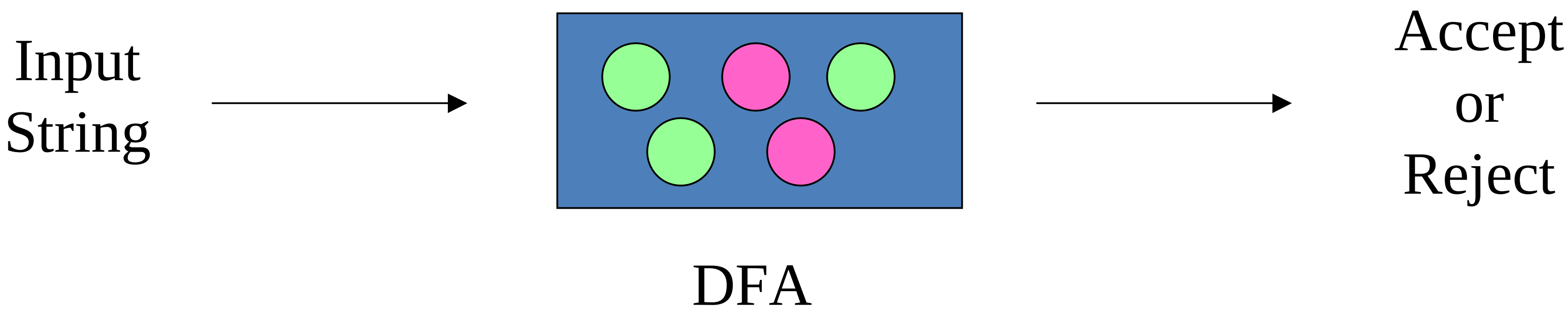
Deterministic Finite Automaton

Deterministic Finite Automaton



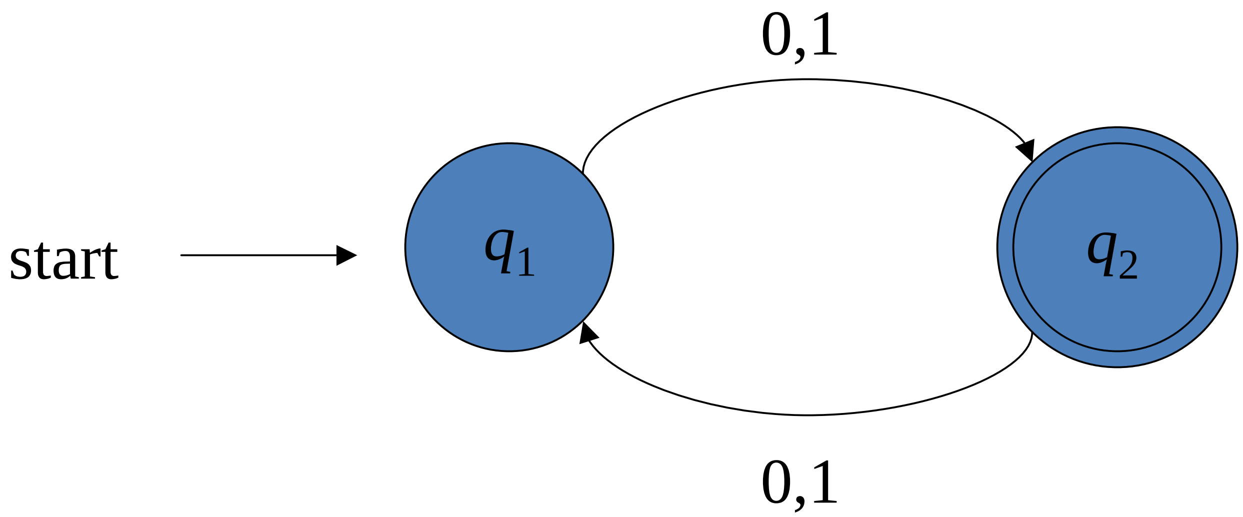
- A machine with finite number of states
- At any time, it is in one of the states
- Initially at start state

Deterministic Finite Automaton



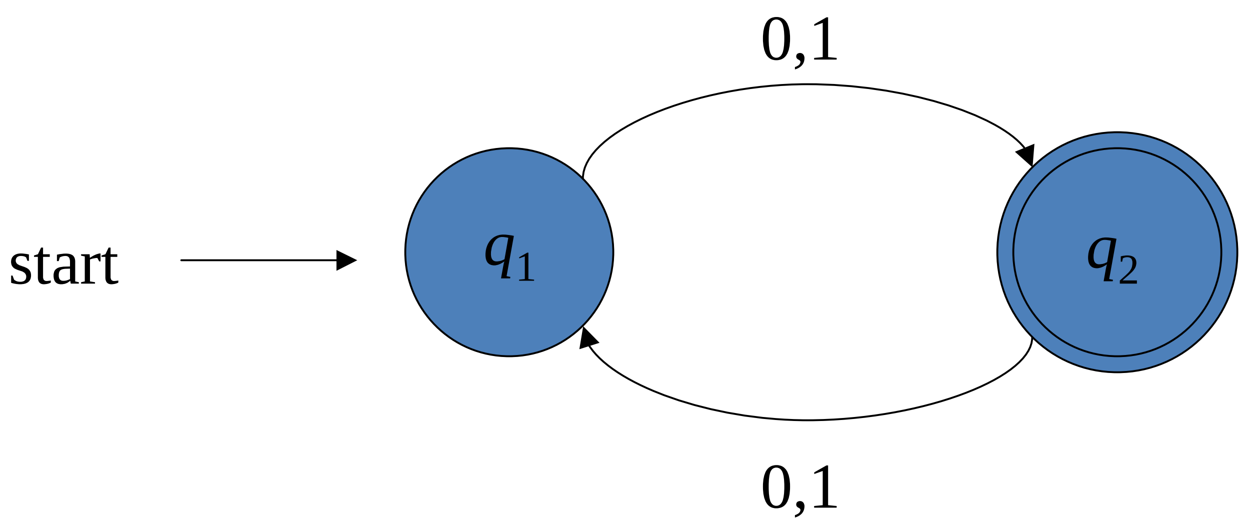
- Reads character of input string one at a time
- Move to next state depending on what is the current state and what is the character read

Example of DFA



- Circles : States (start state = q_1)
- Arrows : How to move between states
- Some states are special : accepting states
  (marked by double circle, such as q_2)

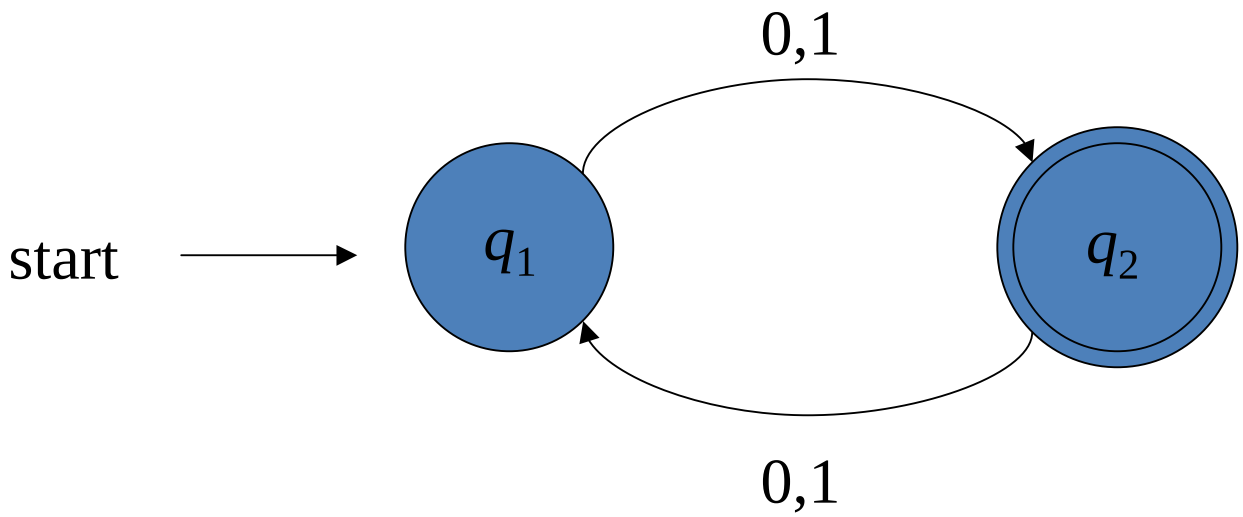
Example of DFA



- If after reading all characters, we land at an accepting state, the string is said to be accepted
  - Else, the string is rejected

Q: What strings are accepted by the above DFA?

Example of DFA



- The DFA accepts exactly strings in

  L = { all binary strings with odd length }

- The set L is called the language of the DFA
  - Note : Each DFA has one language

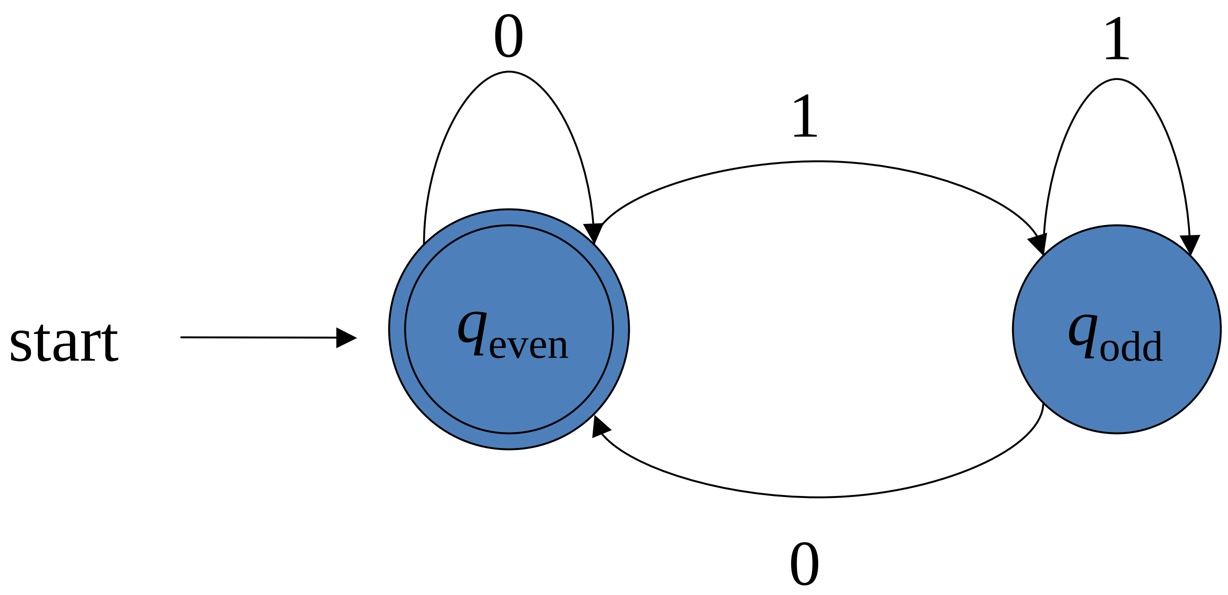
Formal Definition

- A DFA can be described precisely by specifying the following things:

Q	=	the set of states
Σ	=	the set of chars it reads
δ	=	the arrows (transition function)
q_{start}	=	the start state
F	=	the set of accepting states

Designing a DFA

- How to design a DFA which accepts exactly those binary strings representing an even number ?



Designing a DFA

- How to design a DFA which accepts exactly those binary strings representing an integer which is a multiple of five ?

- E.g., it accepts 101 or 1010 or 1111

  but rejects 11 or 001 or 01011

Pumping Lemma

Pumping Lemma

- Suppose a DFA has 5 states, and your friend tells you that this DFA accepts some string of length 10

- Immediately, we can conclude that the language of this DFA is infinite (Why??)

Pumping Lemma (simplified version)

Theorem 1 :

Suppose D is a DFA and L is the language of D.

There is an integer p such that

if D accepts a string longer than p,

then D accepts infinite strings (L is infinite)

The integer p is called the pumping length

Pumping Lemma (stronger version)

Theorem 2 :

Suppose D is a DFA and L is the language of D.

There is an integer p such that

if L contains a string s longer than p,

then s can be divided into $s = xyz$, with

(1) $|y| > 0$, (2) $|xy| \leq p$, and

(3) For each $k \geq 0$, $xy^kz \in L$

Use of Pumping Lemma

- A language L is called regular if L is the language of some DFA
  - Ex : The set of all even length binary strings is a regular language
- Otherwise, L is not the language of any DFA, then L is non-regular

Ex: Show that $\{ \ 0^n 1^n \mid n > 0 \ \}$ is non-regular

Use of Pumping Lemma

Ex : Show that

$\{ \ w \mid w$ has equal number of 0's and 1's $\}$

is non-regular

Ex : Show that $\{ \ 1^x \mid x$ is prime $\}$ is non-regular

Ex: Show that $\{ \ 0^x 1^y \mid x > y \}$ is non-regular