1. Suppose \((G, \star)\) is a group, and both \((H, \star)\) and \((K, \star)\) are subgroups of \((G, \star)\). Show that \((H \cap K, \star)\) is a subgroup of \((G, \star)\).

2. Suppose \((G, \star)\) is a group, and \(e\) be its identity element. For each element \(g\) in \(G\), we define the order of \(g\), \(\text{ord}(g)\), to be the minimum positive integer \(k\) such that \(g^k = e\). For instance, consider the group \((Z_6, \oplus)\), where \(Z_6 = \{0, 1, 2, 3, 4, 5\}\) and \(\oplus\) be the addition under modulo 6. Then the identity element is 0, and the orders of the elements 1, 2, 3, 4, and 5 are respectively as follows:

\[
\text{ord}(1) = 6, \text{ord}(2) = 3, \text{ord}(3) = 2, \text{ord}(4) = 3, \text{ord}(5) = 6.
\]

Show that the order of any element must divide the size of \(G\).

3. A rod is divided into six segments, and each segment will be colored by one of the \(n\) colors. In how many distinct ways can the rod be colored? (For this question, we assume that two colorings are equal if one can be transformed to the other by rotating the rod by 180°.)

4. Design a finite automaton that accepts exactly all binary strings each of which ends with 11. For instance, the automaton should accept 11, 0111, but should reject 1, 0110, or the empty string.

5. A palindrome is a string that reads the same forward and backward. For instance, 010 and 1001 are palindromes, but 1010 is not. Show that the language

\[ L = \{ w \mid w \text{ is a binary string and } w \text{ is a palindrome} \} \]

is non-regular.

6. (Just for fun: No marks) Suppose \((G, \star)\) is a group, and both \((H, \star)\) and \((K, \star)\) are subgroups of \((G, \star)\). Define \(HK\) to be the set \(\{hk \mid h \in H \text{ and } k \in K\}\), and define \(KH\) similarly.

Show that \((HK, \star)\) is a subgroup of \((G, \star)\) if and only if \(HK = KH\).

7. (Just for fun: No marks) Let \(D\) be the set of binary strings where each string contains equal number of substrings 01 and 10.

For instance, 101 is in \(D\) since it has one substrings of 01 and one substrings of 10. Similarly, 1111 is also in \(D\) since it does not have any substrings of 01 and 10. However, 1010 is not in \(D\), as it contains one 01 but two 10s.

Show that \(D\) is a regular language.