CS2335 Special Topics in Discrete Structure

Homework 6

Due: 1:10 pm, January 07, 2010 (before class)

1. Suppose $(G, \star)$ is a group, and both $(H, \star)$ and $(K, \star)$ are subgroups of $(G, \star)$.

   Show that $(H \cap K, \star)$ is a subgroup of $(G, \star)$.

2. Suppose $(G, \star)$ is a group, and e be its identity element. For each element g in G, we define the *order* of g, $ord(g)$, to be the minimum positive integer k such that $g^k = e$.

   For instance, consider the group $(Z_6, \oplus)$, where $Z_n = \{0, 1, 2, 3, 4, 5\}$ and $\oplus$ be the addition under modulo 6. Then the identity element is 0, and the orders of the elements 1, 2, 3, 4, and 5 are respectively as follows:

   $$ord(1) = 6,\ ord(2) = 3,\ ord(3) = 2,\ ord(4) = 3,\ ord(5) = 6.$$

   Show that the order of any element must divide the size of G.

3. A rod is divided into six segments, and each segment will be colored by one of the n colors. In how many distinct ways can the rod be colored? (For this question, we assume that two colorings are equal if one can be transformed to the other by rotating the rod by 180°.)

4. Design a finite automaton that accepts exactly all binary strings each of which ends with `11`. For instance, the automaton should accept `11`, `0111`, but should reject `1`, `0110`, or the empty string.

5. A *palindrome* is a string that reads the same forward and backward. For instance, `010` and `1001` are palindromes, but `1010` is not. Show that the language

   $$L = \{w \mid \ w \text{ is a binary string and } w \text{ is a palindrome}\}$$

   is non-regular.

6. (Just for fun: No marks) Suppose $(G, \star)$ is a group, and both $(H, \star)$ and $(K, \star)$ are subgroups of $(G, \star)$. Define HK to be the set $\{hk \mid h \in H \text{ and } k \in K\}$, and define KH similarly.

   Show that $(HK, \star)$ is a subgroup of $(G, \star)$ if and only if $HK = KH$.

7. (Just for fun: No marks) Let D be the set of binary strings where each string contains equal number of substrings `01` and `10`.

   For instance, `101` is in D since it has one substrings of `01` and one substrings of `10`. Similarly, `1111` is also in D since it does not have any substrings of `01` and `10`. However, `1010` is not in D, as it contains one `01` but two `10`s.

   Show that D is a regular language.