# CS2335 SPECIAL TOPICS IN DISCRETE STRUCTURE

Homework 3

Due: 1:10 pm, November 23, 2009 (before class)

1. Solve the following recurrence relations.

   (a) $\begin{cases} a_n = 5a_{n-1} + 6a_{n-2}, & n \geq 2 \\ a_0 = 1 \\ a_1 = 3 \end{cases}$

   (b) $\begin{cases} 2a_{n+2} - 11a_{n+1} + 5a_n = 0, & n \geq 0 \\ a_0 = 2 \\ a_1 = -8 \end{cases}$

   (c) $\begin{cases} a_n - 6a_{n-1} + 9a_{n-2} = 0, & n \geq 2 \\ a_0 = 5 \\ a_1 = 12 \end{cases}$

2. Solve the following recurrence relations.

   (a) $\begin{cases} a_{n+1} - a_n = 2n + 3, & n \geq 0 \\ a_0 = 1 \end{cases}$

   (b) $\begin{cases} a_{n+1} - a_n = 2n^2 - n, & n \geq 0 \\ a_0 = 3 \end{cases}$

   (c) $\begin{cases} a_{n+1} - 2a_n = 2^n, & n \geq 0 \\ a_0 = 1 \end{cases}$

3. Solve the following recurrence relations by the method of generating functions.

   (a) $\begin{cases} a_{n+2} - 2a_{n+1} + a_n = 2^n, & n \geq 0 \\ a_0 = 1 \\ a_1 = 2 \end{cases}$

   (b) $\begin{cases} a_{n+1} = 2a_n - b_n + 2, & n \geq 0 \\ b_{n+1} = -a_n + 2b_n - 1, & n \geq 0 \\ a_0 = 0 \\ b_0 = 1 \end{cases}$

4. Let $a_n$ denote the number of $n$-bit binary strings in which the pattern 111 occurs exactly twice, with the second occurrence at the $n$th bit. Find the generating function for $(a_0, a_1, \ldots)$.

5. (a) Let $b_n$ denote the number of $n$-bit binary strings in which the pattern 11011 *first* occurs at the $n$th bit. Find the generating function for $(b_0, b_1, \ldots)$.

   (b) Let $c_n$ denote the number of $n$-bit binary strings in which the pattern 11011 occurs at the $n$th bit. Using the result of part (a), find the generating function for $(c_0, c_1, \ldots)$.

6. Let $d_n$ be the number of ways to completely cover a $2 \times n$ rectangle with $2 \times 1$ dominoes. Express $d_n$ in terms of $n$. (Show your steps.)