

CS2335 SPECIAL TOPICS IN DISCRETE STRUCTURE

Homework 2

Due: 1:10 pm, October 22, 2009 (before class)

1. Suppose m and n are non-negative, and $k \leq \min(m, n)$. What is the following sum? (Explain why your sum is correct.)

$$\binom{n}{0}\binom{m}{k} + \binom{n}{1}\binom{m}{k-1} + \cdots + \binom{n}{k}\binom{m}{0}$$

2. Find the value of a_{50} in the following expansion:

$$\frac{x-3}{x^2-3x+2} = a_0 + a_1x + a_2x^2 + \cdots + a_{50}x^{50} + \cdots$$

3. In how many ways can $3n$ letters be selected from $2n$ A's, $2n$ B's, and $2n$ C's? (Order of letters is not important.)
4. Find the number of n -digit strings generated from the alphabet $\{0, 1, 2, 3, 4\}$ whose number of 0's and number of 1's are both even.
5. Find the number of n -digit strings generated from the alphabet $\{0, 1, 2, 3, 4\}$ whose *total number* of 0's and 1's is even.
6. Find the ordinary generating function of the sequence (a_0, a_1, a_2, \dots) where a_r is the number of partitions of the integer r into distinct primes.
7. (Challenging: No marks) Suppose n and k are non-negative, with $k \leq n$. What is the following sum? (Explain why your sum is correct.)

$$\binom{n}{0}\binom{n}{k} - \binom{n}{1}\binom{n-1}{k-1} + \cdots + (-1)^k\binom{n}{k}\binom{n-k}{0}$$

8. (Challenging: No marks) Find the exponential generating function of the sequence:

$$(1, 1 \times 4, 1 \times 4 \times 7, \dots, 1 \times 4 \times \cdots \times (3r+1), \dots)$$

9. (Challenging: No marks)

(a) Evaluate the definite integral

$$\int_0^\infty e^{-s} s^k ds.$$

(b) Let $A(x)$ and $E(x)$ be the ordinary and exponential generating functions of the sequence of numbers (a_0, a_1, a_2, \dots) , respectively. Show that

$$A(x) = \int_0^\infty e^{-s} E(sx) ds.$$