CS5371 Theory of Computation

Homework 3
Due: 2:10 pm, November 28, 2006 (before class)

1. Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

2. Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A.[†] (Hint: You may find it helpful to consider an enumerator for A, and re-visit the diagonalization technique.)

3. Let $E = \{\langle M\rangle \mid M$ is a DFA that accepts some string with more 1s than 0s$\}$. Show that E is decidable. (Hint: Theorems about CFLs are helpful here.)

4. Let C be a language. Prove that C is Turing-recognizable if and only if a decidable language D exists such that $C = \{x \mid \exists y(\langle x, y\rangle \in D)\}$.

5. (Bonus Question) Show that the problem of determining whether a CFG generates all string in 1^* is decidable. In other words, show that $\{\langle G\rangle \mid G$ is a CFG over $\{0, 1\}$ and $1^* \subseteq L(G)\}$ is a decidable language.

[†]The question seems strange at the first glance. In fact, it is asking you to prove that the language consisting of *all* descriptions of Turing deciders is not Turing-recognizable.