CS4311
Design and Analysis of Algorithms

Tutorial: Hints on Assignment 4

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Outline

- Description + Hints for Assignment 4

Question 1

- We have a lot of sushi in a table.
- They are arranged into a line.

many kinds of sushi

Each sushi has a quality value (higher if it tastes better)
The quality values of sushi are distinct.

Question 1

- Find a way to take sushi such that

  (1) total quality value is maximized, and

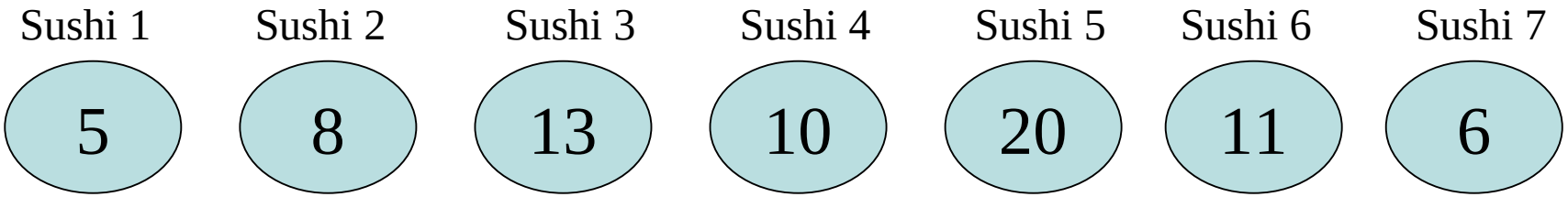
  (2) the quality value of sushi taken is increasing

  i.e. when you take a sushi A, you must next take a sushi B better than A.

- Your algorithm should run in $O(n^2)$ time

Hint: Suppose you know the best way to take the first k sushi, for each k = 1, 2, …, n-1.
Can you tell the best way to take the n sushi ?

Example



The following are some possible ways to take the sushi.

(1) 5, 8, 10, 11.

(2) 5, 8, 13, 20.

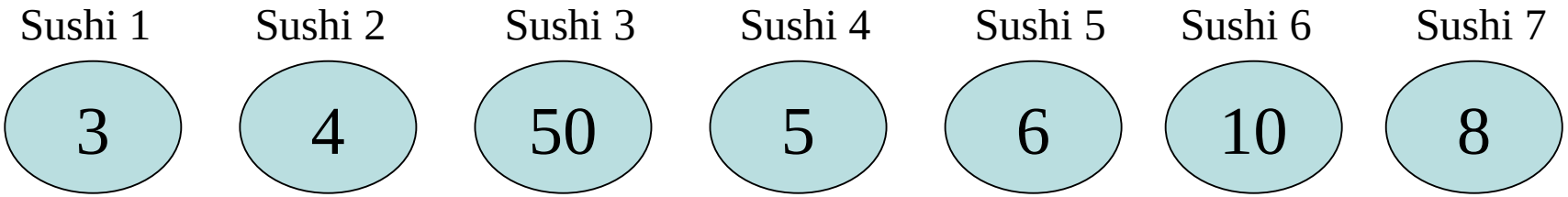
(3) 5, 8, 10, 20.

(4) 8, 13.

(5) …

It turns out (2) is the maximum among all possible ways.

Example



The following are some possible ways to take the sushi.

(1) 3, 4, 50.

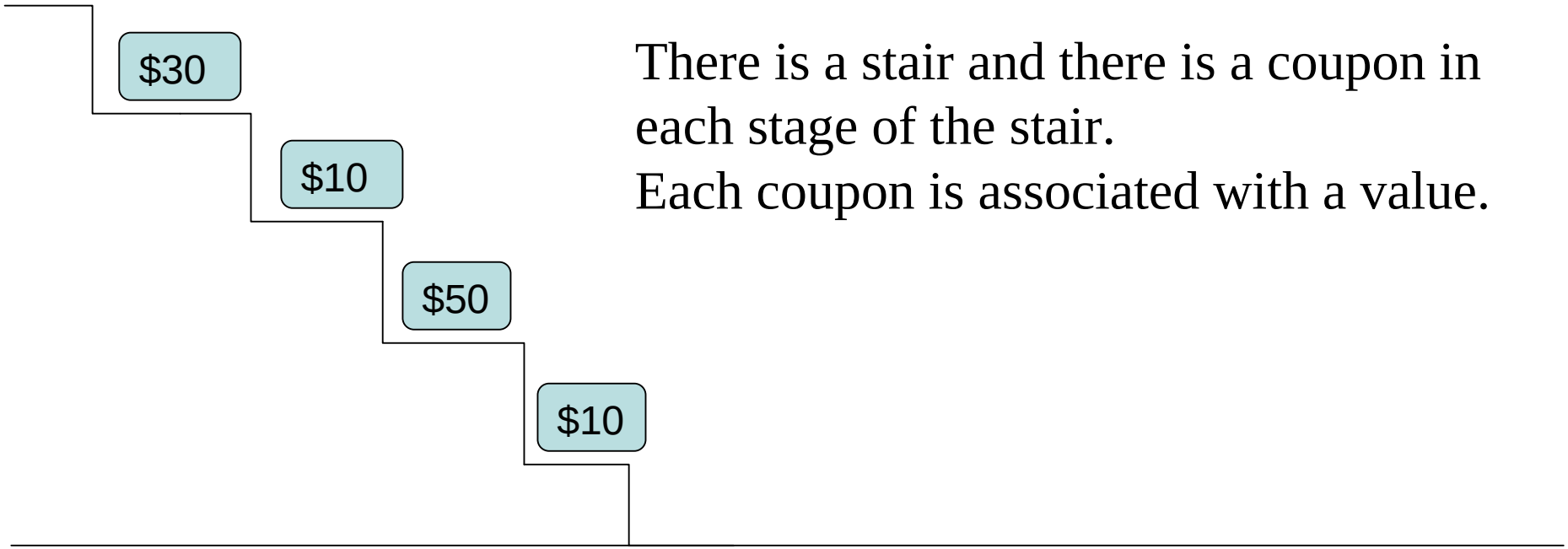
(2) 3, 4, 5, 6, 10.

(3) 3, 4, 5, 6, 8.

(4) …

It turns out (1) is the maximum among all possible ways.

Question 2



There is a stair and there is a coupon in each stage of the stair.
Each coupon is associated with a value.

You need to climb to the top in at most L steps.
Since you are in a hurry, you need to climb 1, 2 or 3 stages (upwards) in each step.
When at a stage, you could collect the coupon on that stage.

Question 2

- Find a way to climb the stair and pick up the maximum value of the coupons in $O(n^2)$ time.

- Hint: Suppose you know the best way to climb to stage k, using **exactly** X steps, for each k = 1, 2, …, n-1.

  Do you know the best way to climb to stage k+1, using **exactly** X+1 steps ?

Example

s0	s1	s2	s3	s4	s5	s6
	$30	$40	$20	$10	$30	

You want to climb to stage 6 in at most 3 steps.

Then the answer should be s2, s5, s6.

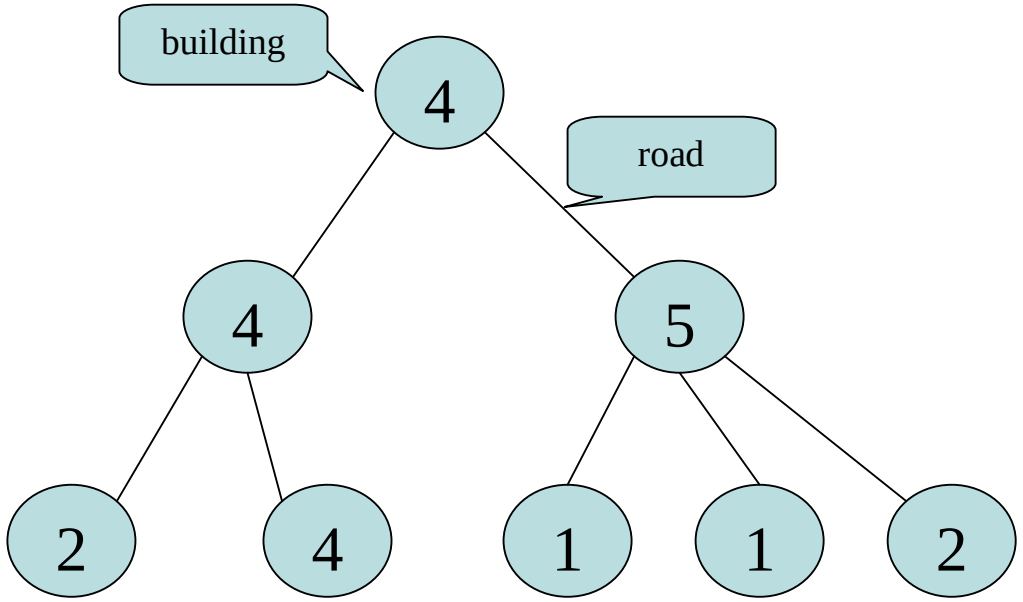
Example

s0	s1	s2	s3	s4	s5	s6
	$30	$40	$80	$10	$30	

You want to climb to stage 6 in at most 3 steps.

Then the answer should be s2, s3, s6.

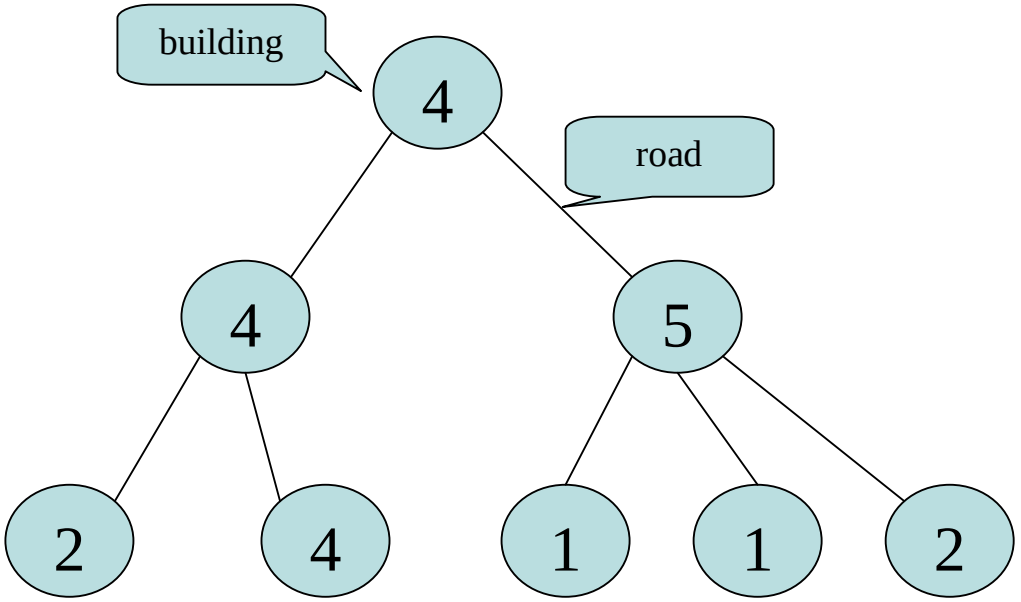
Question 3



There is a city which looks like a tree, with
buildings = nodes, roads = edges.

We want to open stinky-tofu stores in the city.

Question 3



Each building can open a store, and it has a value representing # customers who will visit the building.

Each store cannot be adjacent to each other (i.e., stores cannot be located on the same edge).

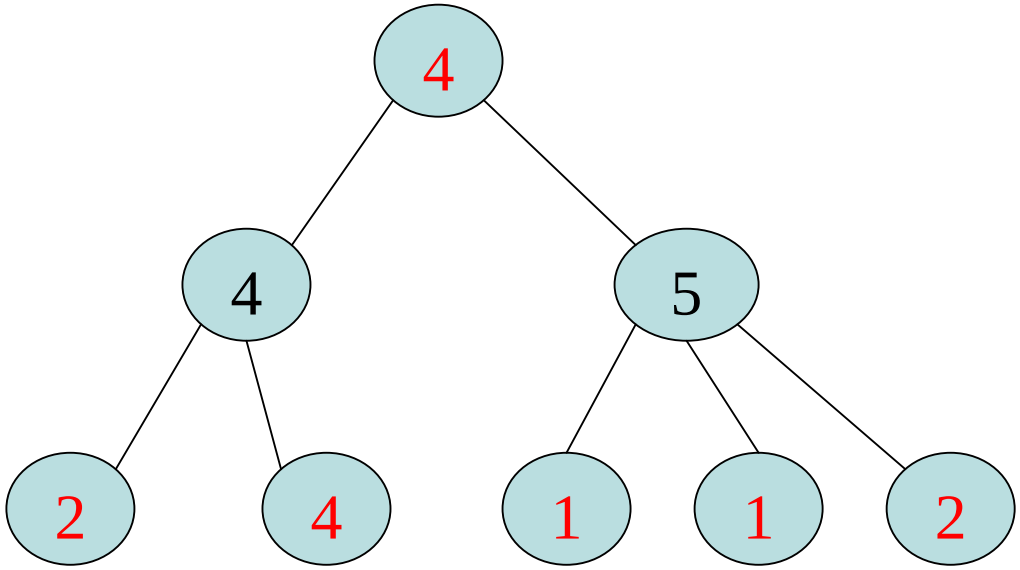
Question 3

Target: Open the stores so as to maximum the total # customers

Your algorithm should run in O(n) time.

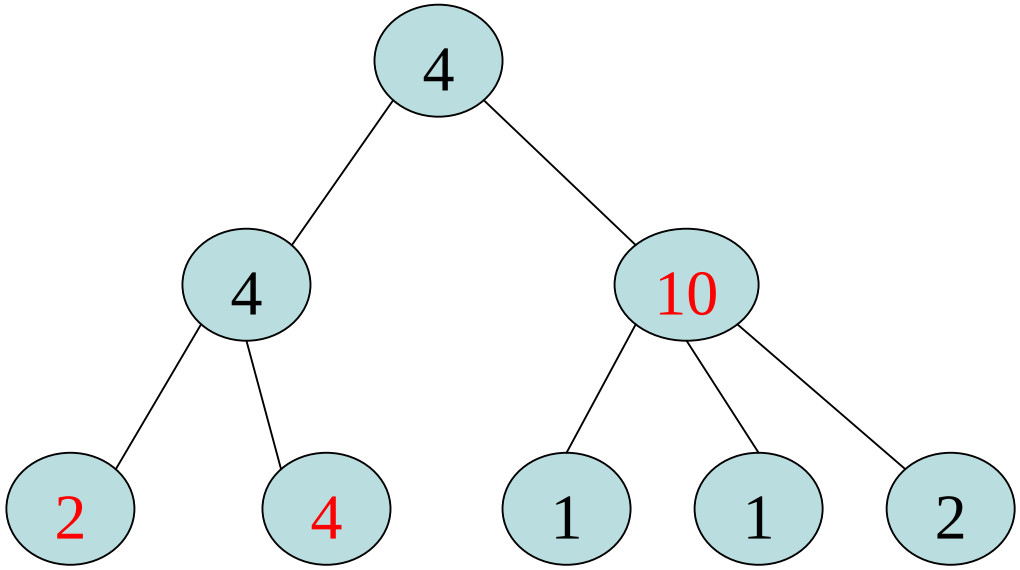
Hint: Bottom-up approach.

Example



In this tree, we should open the stinky-tofu stores in the buildings with red numbers.

Example



In this tree, we should open the stinky-tofu stores in the buildings with red numbers.

Question 4

Question 4

- Each dish has a value and its volume, where each volume is an integer.
- Your stomach has a capacity of V units, where V is an integer.
- Find a way to eat dishes with the maximum total value such that your stomach could hold.

- Your algorithm should run in O(nV) time.
- Hint: optimal substructure.

Example

Dish	Orange	Beef	Vegetable	Cake	Pudding	Apple	Tofu	Egg	Chicken
$$	30	100	10	40	15	35	10	10	80
volume	15	40	20	10	10	15	30	8	60

Assume V=80

The answer should be beef + cake + orange + apple.

Thank You