

CS4311 DESIGN AND ANALYSIS OF ALGORITHMS

Homework 5

Due: 10:10 am, May 19, 2009 (before class)

1. (20%) One day a camel asks for your help. He wants to make a trip from Egypt to Turkey, but he can only travel 100km with his stomach filled with bananas. There are several oases on the road which supply bananas, but he wants to visit as few oases as possible. Suppose you already know the position of oases on the road from Egypt to Turkey, and assume that there is at least a way for the camel to travel from Egypt to Turkey. Design a greedy algorithm to help the poor camel finish his trip.

Requirements:

- (a) The camel must visit minimum oases with your algorithm.
 - (b) The algorithm must be a greedy algorithm (with a brief proof).
 - (c) The camel must arrive Turkey.
2. (60%) Bill has invented a strange data structure called *Flipping Stack*. Flipping Stack supports only FLIPPING-PUSH() function. In each FLIPPING-PUSH(), an item is first pushed to the stack, and check if the number of items in the stack is a power of 2 (i.e., equals to 2^i for some i). If so, all items in the stack will have to be flipped. For instance, suppose we use FLIPPING-PUSH() to push the items 1, 2, 3, 4 into the stack, the contents of the stack (viewed from bottom to top) after each push are as follows:

$$(1) \Rightarrow (2, 1) \Rightarrow (2, 1, 3) \Rightarrow (4, 3, 1, 2)$$

Bill asks you to analyze the amortized cost of FLIPPING-PUSH.

Requirements:

- (a) The cost for flipping of the stack is equal to the number of items currently in the stack.
 - (b) Analyze the amortized cost with all the three methods taught in class. (Aggregate, Accounting, Potential)
3. (20%) A min-heap with n elements supports INSERT and EXTRACT-MIN in $O(\log n)$ worst-case time. Give a potential function Φ such that the amortized cost of INSERT is $O(\log n)$ and the amortized cost of EXTRACT-MIN is $O(1)$.

Requirements:

- (a) You have to describe your potential function Φ clearly.
- (b) You must show that your potential function works.

(5%) Explain why it is impossible to have the amortized cost of INSERT to be $o(\log n)$ and at the same time the amortized cost of EXTRACT-MIN is $O(1)$.

(Hint: Sorting lower bound.)