CS4311 Design and Analysis of Algorithms

Homework 4
Due: 10:10 am, April 28, 2009 (before class)

1. Peter is an owner of a Japanese sushi restaurant and today he invites you for dinner at his restaurant. In front of you are n sushi dishes that are arranged in a line. Each dish are different and they have different costs.

   Peter hopes that you can select the dishes you want, starting from the left to the right. However, there is a further restriction: when you select a dish, say A, the next dish you can select must cost higher than A.

   (25%) Design an $O(n^2)$-time algorithm to select the dishes so as to maximize the total costs.

2. There is a stair with n stages, and on each stage there is a coupon to your favorite restaurant. Each coupon has an associated value which may be different.

   You can climb up 1, 2, or 3 stages in a step. Since you are in a hurry, you need to reach the top of the stair in at most L steps, where $L \leq n$. When you visit a particular stage, you can collect the coupon on that stage.

   (25%) Find a way to climb the stair within L steps so that you can collect coupons with maximum total value. Your algorithm should run in $O(n^2)$ time.

3. You have newly obtained the right to invest at a strange city called `Trellisland`. The shape of the city looks like a tree, such that each node in the tree has a building, and each edge in the tree is a road connecting two buildings. And in total, there are n buildings.

   You want to open stinky tofu stores in this city. According to the rules, each building can have at most one store, and each store cannot be adjacent to each other on the same road (otherwise, it becomes *too* stinky).

   You have done some preliminary study, and can now tell the number of customers who will visit each building if a store is open there. Now, your target is to decide where to open the stores so as to maximize the total number of customers.

   (25%) Design an $O(n)$-time algorithm to achieve the above target.

4. After a lot of hard work, you have successfully opened the stinky tofu stores. In order to celebrate, you go to a buffet restaurant to have a big dinner.

   There are n distinct dishes, each has a different volume (which is an integer). It is obvious that your stomach capacity, V (which is also an integer), is not large enough to hold all the dishes. Also, some dish looks expensive and some looks so-so, so that each dish has a different value to you.

   Now, you have decided the following strategy: (i) Never take the same dish twice, and (ii) do not overeat (so that total volume is at most V).

   (25%) Design an $O(nV)$-time algorithm to select the dishes so as to maximize the total values.