CS4311 Design and Analysis of Algorithms

Homework 3

Due: 10:10 am, March 31, 2008 (before class)

1. Let A be an array of n distinct numbers. In the lecture, we have learnt that finding the kth smallest number of A, for any k, can be done in $O(n)$ time.

   Suppose that we now want to know more about the array. In particular, we want to find out the 2^jth smallest numbers of A, for all $j = 0, 1, \ldots, \lfloor \log n \rfloor$. (That is, we simultaneously want know which numbers are respectively the 1st, the 2nd, the 4th, the 8th, ... smallest numbers of A.)

   (25%) Design an $O(n)$-time algorithm to accomplish the above task.

2. Similar to Question 1, but this time we want to get the first $\sqrt{n}$ smallest numbers of A in sorted order.

   (25%) Show that the above can also be done in $O(n)$ time.

3. Similar to Question 1, but this time we want to get the k numbers which are closest to the median. However, we are not required to sort these k numbers when we output them.

   For example, suppose the array A is as follows:

   $$A = \langle 2.3, 0.1, 2.1, 1.6, 0.5, 3.7, 2.2 \rangle.$$

   Then, the median of the array is 2.1, and the 3 numbers closest to the median would be $2.1, 2.2$, and 2.3.

   (25%) Design an $O(n)$-time algorithm to accomplish this task. Note that your algorithm needs to work for any value of k.[†]

4. Let B be a set of n integers, with each integer taking a value between 0 and n^2.

   (25%) Show that the integers in B can be sorted in $O(n)$ time.

5. (Just for fun. No marks) Your friend, Peter, is an owner of 25 horses, and he wants to find out the three fastest horses among these 25 horses.

   Unfortunately, Peter doesn't have any timing device. All he has is a racecourse that allows a competition of five horses each time. In particular, if any five horses are placed in the racecourse for a trial, then we will be able to tell the relative speed of these five horses.

   As a friend of Peter, you want to help him to minimize the number of trials needed to determine the best three horses. Can you design a racing schedule which requires at most 8 trials? How about 7 trials?[‡]

[†]This question is a bit difficult; try to spend some more time on it. Add oil!

[‡]This question was once asked during Wing-Kai's job interview ^o^;