Tail Recursion

Speaker : MARK

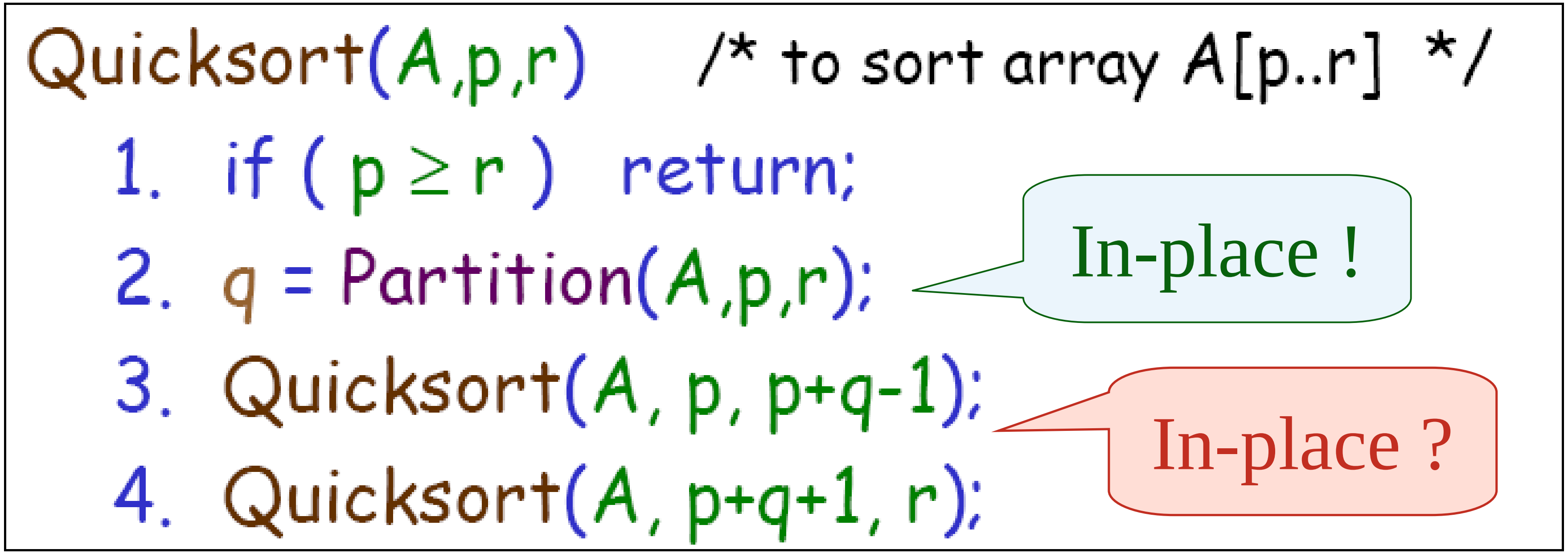
What is in-place algorithm?

- Algorithm that uses O(1) extra space in addition to the original input

- How about Quicksort ?
  - Quicksort has in-place partition
  - Then, Quicksort is in-place algorithm ? NO !!

Quicksort

The Quicksort algorithm works as follows:

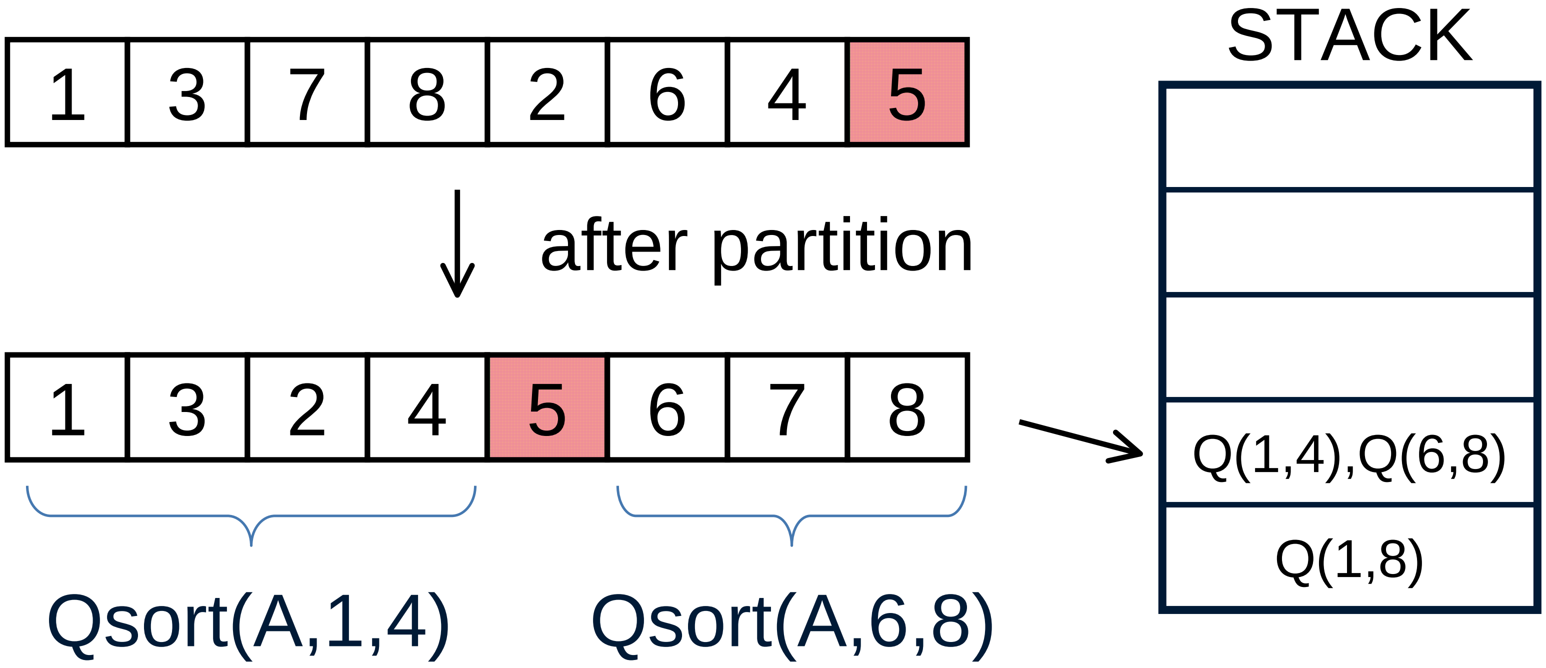


```
Quicksort(A,p,r)     /* to sort array A[p..r] */
    if ( p ≥ r )  return;
    q = Partition(A,p,r);
    Quicksort(A, p, p+q-1);
    Quicksort(A, p+q+1, r);
```

In-place !

In-place ?

Quicksort needs stack



1	3	7	8	2	6	4	5

after partition

1	3	2	4	5	6	7	8

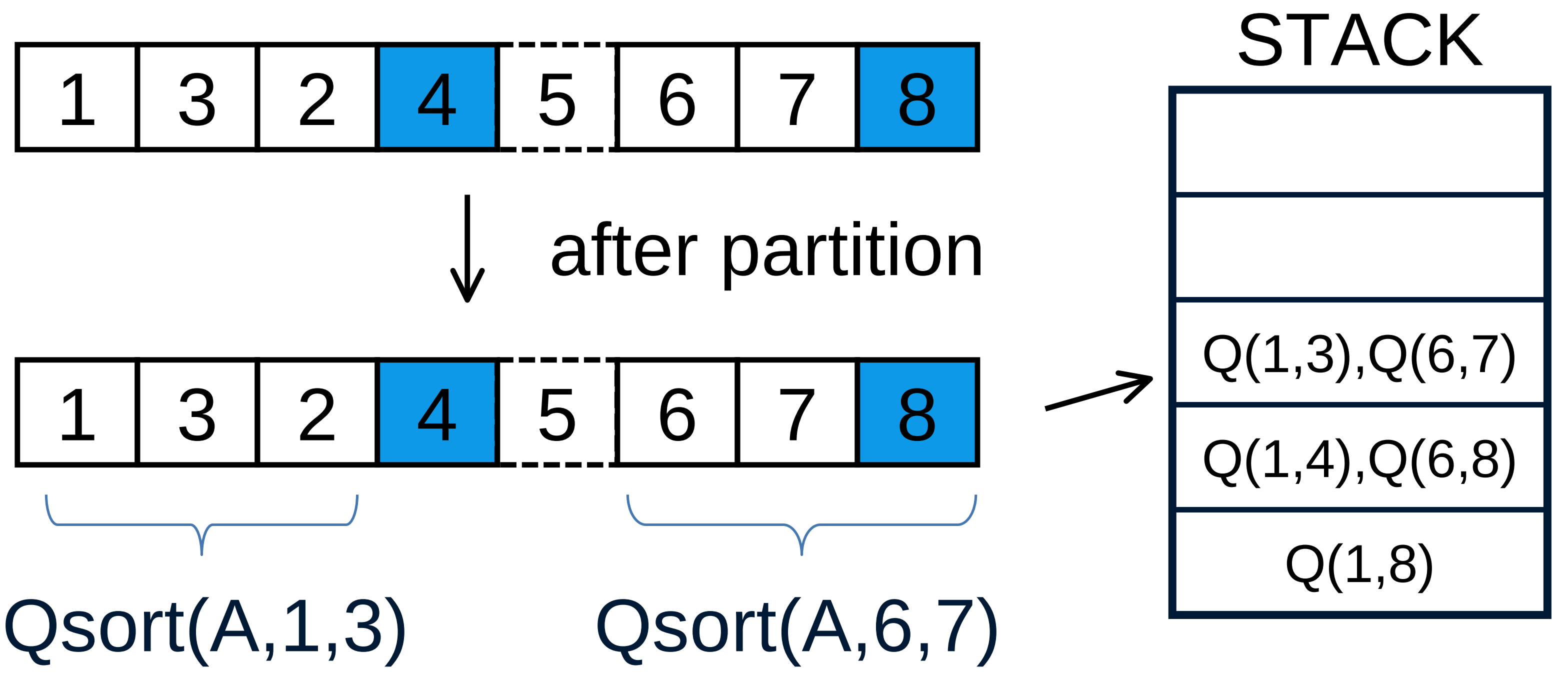
Qsort(A,1,4)

Qsort(A,6,8)

STACK

Q(1,4),Q(6,8)
Q(1,8)

Quicksort needs stack (cont.)



1	3	2	4	5	6	7	8

after partition

1	3	2	4	5	6	7	8

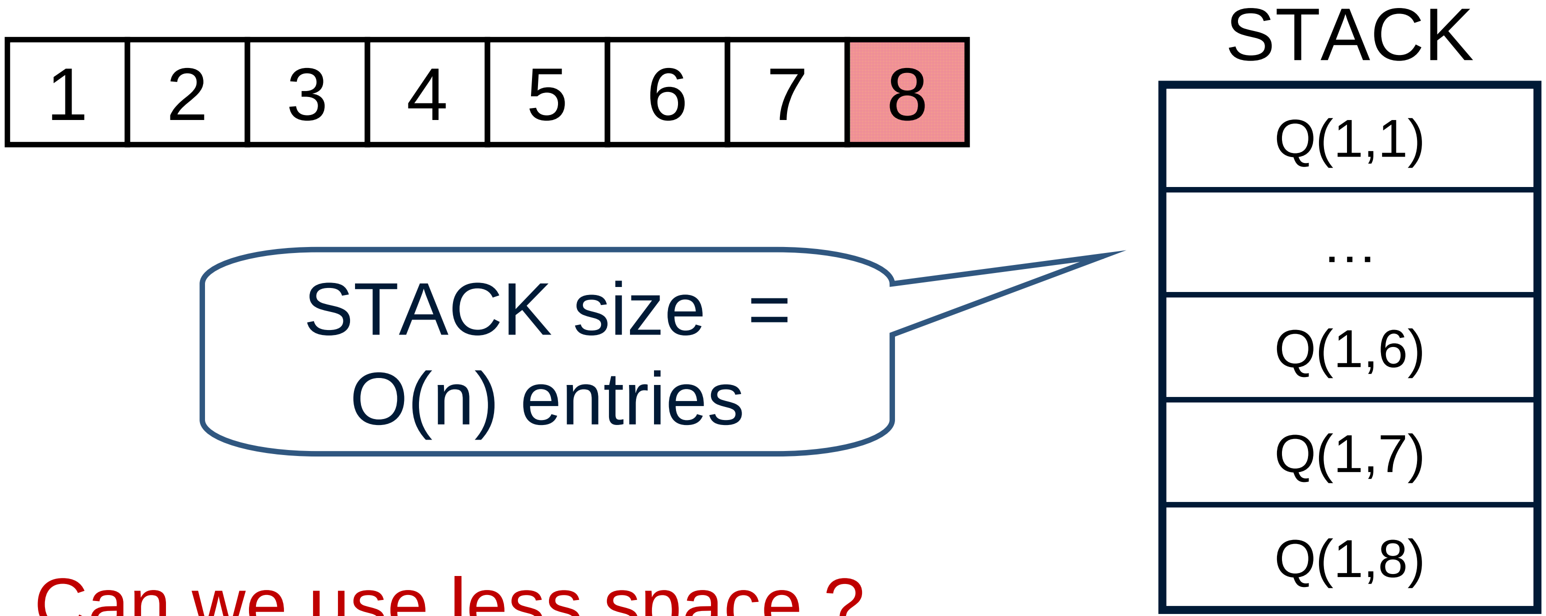
Qsort(A,1,3)

Qsort(A,6,7)

STACK

Q(1,3),Q(6,7)
Q(1,4),Q(6,8)
Q(1,8)

Worst Case Space



1	2	3	4	5	6	7	8

STACK

Q(1,1)

…

Q(1,6)

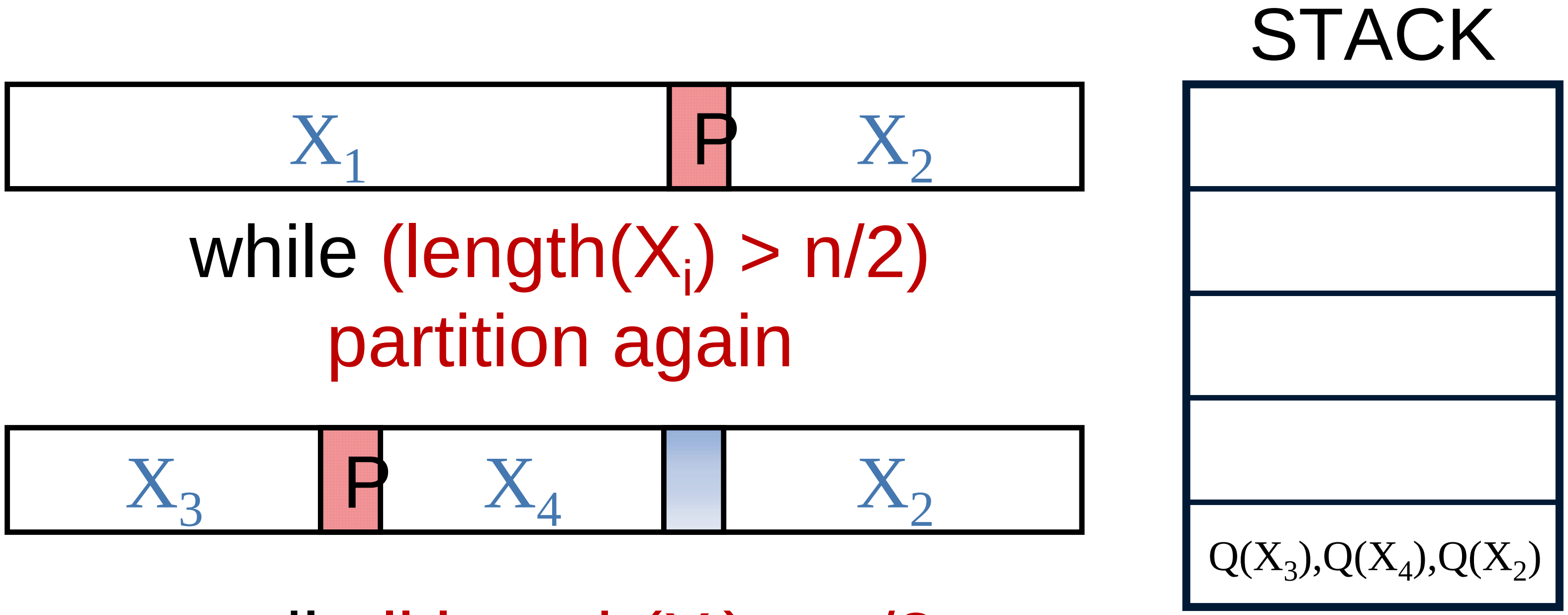
Q(1,7)

Q(1,8)

STACK size =
O(n) entries

Can we use less space ?

Method I

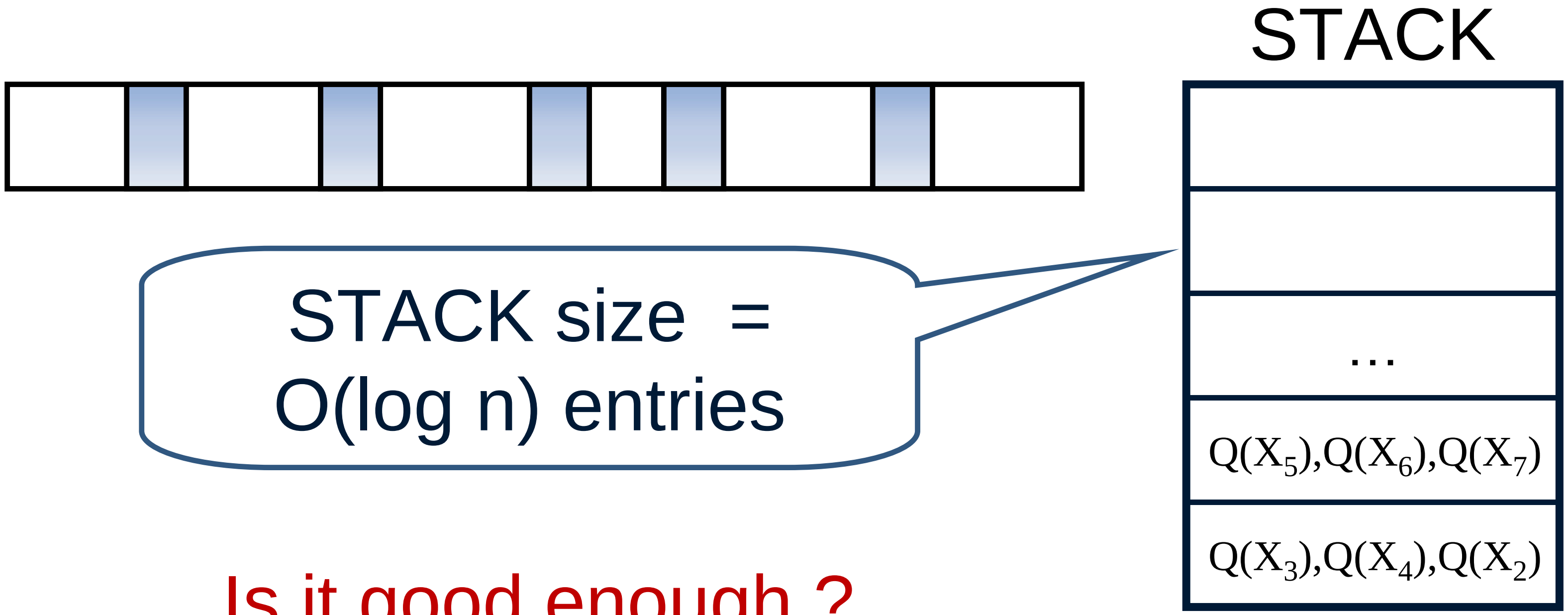


while $(\text{length}(X_i) > n/2)$

partition again

until all $\text{length}(X_i) < n/2$

Method I (cont.)



STACK

STACK size = O(log n) entries

…

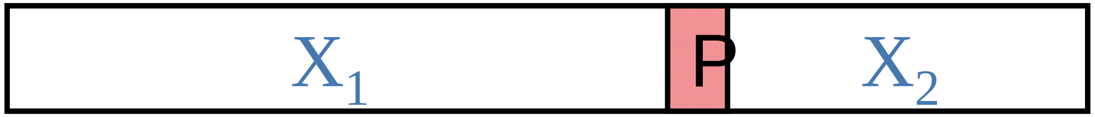
$Q(X_5),Q(X_6),Q(X_7)$

$Q(X_3),Q(X_4),Q(X_2)$

Is it good enough ?

No! Space of an entry may be as large as O(n)

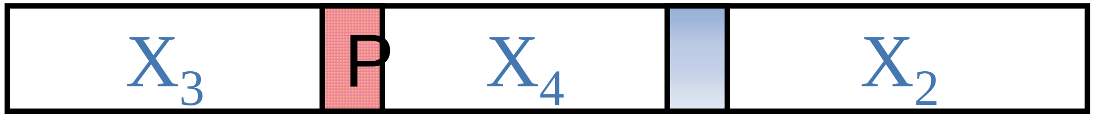
Method II

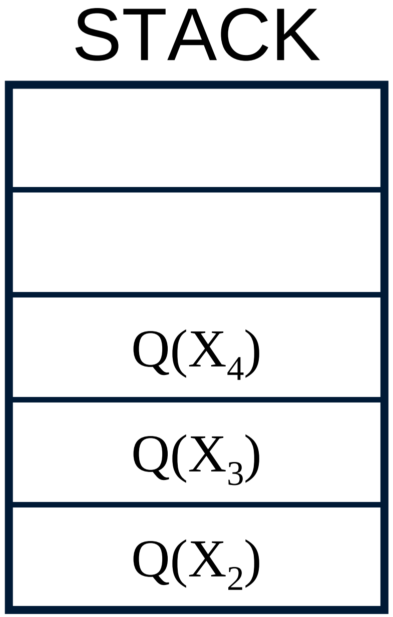


if (there is X with length(X) < n/2)

        call Qsort(X)

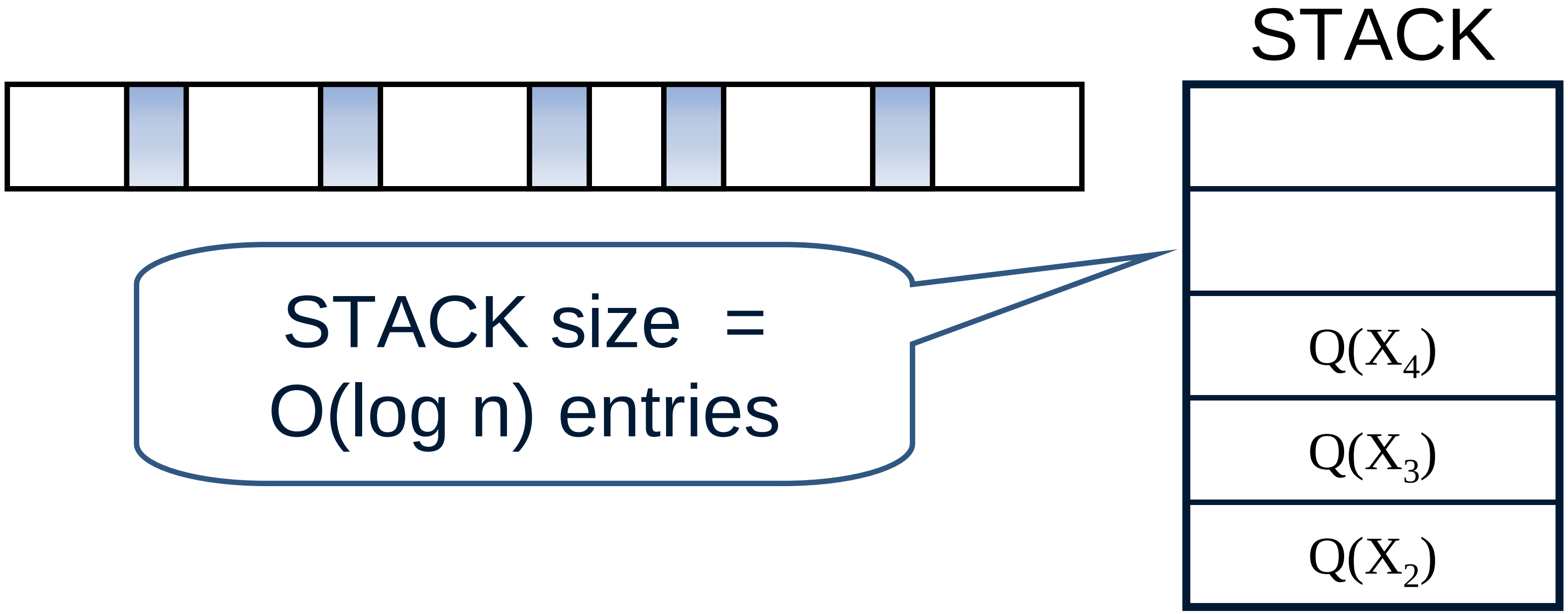
else partition X into X' and X''





until all X are processed

Method II (cont.)



STACK size =
O(log n) entries

Space of every entry is only O(1)

Conclusion

- The idea of Method II is tail recursion
  - First solves sub-problem with smaller size
  - Call recursion only when sub-problem is small enough
- Even with the improvement, Method II 's space complexity = input + O(log n)
  - Still not in-place algorithm !!