Design and Analysis of Algorithms

Homework 1

Outline

• Question 1	Basic
• Question 2	Basic
• Question 3	Basic
• Question 4	Moderate
• Question 5	Moderate
• Question 6(a) and 6(b)	Challenge

Question 1

- Practice to solve the following recurrences:
- (a) $T(n) = 9T(n/2) + n^3$
- (b) $T(n) = 7T(n/2) + n^3$
- (c) $T(n) = T(\sqrt{n}) + \log n$
- (d) $T(n) = 0.5T(n/2) + n$
- (e) $T(n) = 3T(n/3) + n/3$

Question 1

- You can solve by substitution method, recursion tree method or master theorem
- T(1) = 1 (if necessary)
- Do your best to use Θ-notation. If you can not use Θ-notation, then use O-notation.

Question 2

- Prove that

  if $f(n) \in \omega(g(n))$, then $f(n) \notin O(g(n))$

- Hint: $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

Question 3

Merge Sort

- Recall that it first divides the list into two parts

3	1	8	7	6	5	4	2

- Then sorts each part recursively

3	1	8	7	6	5	4	2

- Merges the two sorted parts
- Running time: $\Theta(n \log n)$

Question 3

Consider another merge sort

- It first divides the list into three parts

3	1	8	7	6	5	4	2	9

- Then sorts each part recursively

3	1	8	7	6	5	4	2	9

- Merges the three sorted parts
- Running time: ?

Question 3

- How to merge the three sorted parts together?
- Is the three-part merge sort faster than the two-part merge sort? Why?
  - compare by asymptotic notation

Question 4

- Analyze the running time of the following code

```
for (i = 1; i <= n; i++) {
        for (j = 1; j <= n; j += i)
                x = x + 1;
}
```

- Please use Θ-notation

Question 5

- Your smart friend discovers a new sorting algorithm based on a function called **Greedy Pick** described as follow

Input:	an integer sequence A
Method:	Pick an increasing sequence start from the first element of A
Output:	the increasing sequence

Question 5

- Greedy pick example:

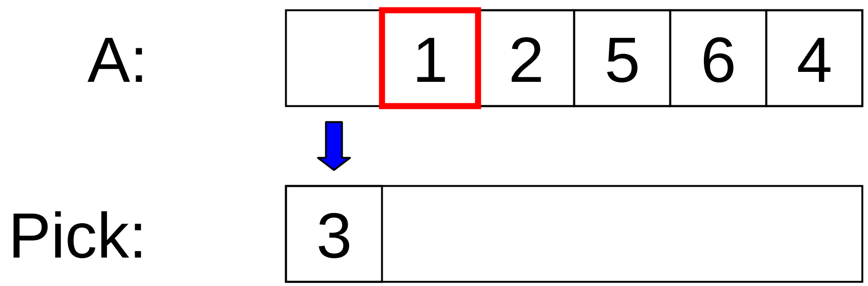
A:

3	1	2	5	6	4

Pick:

Question 5

- Greedy pick example:



Question 5

- Greedy pick example:

A:		1	2	5	6	4
Pick:	3					

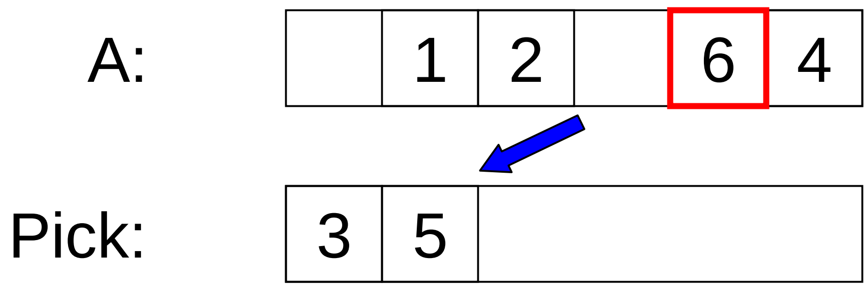
Question 5

- Greedy pick example:

A:		1	2	5	6	4
Pick:	3					

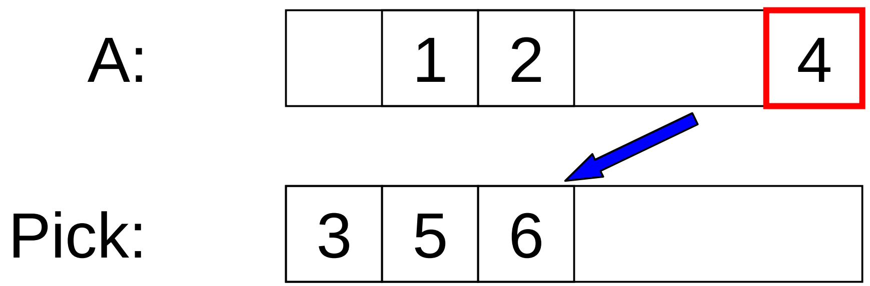
Question 5

- Greedy pick example:



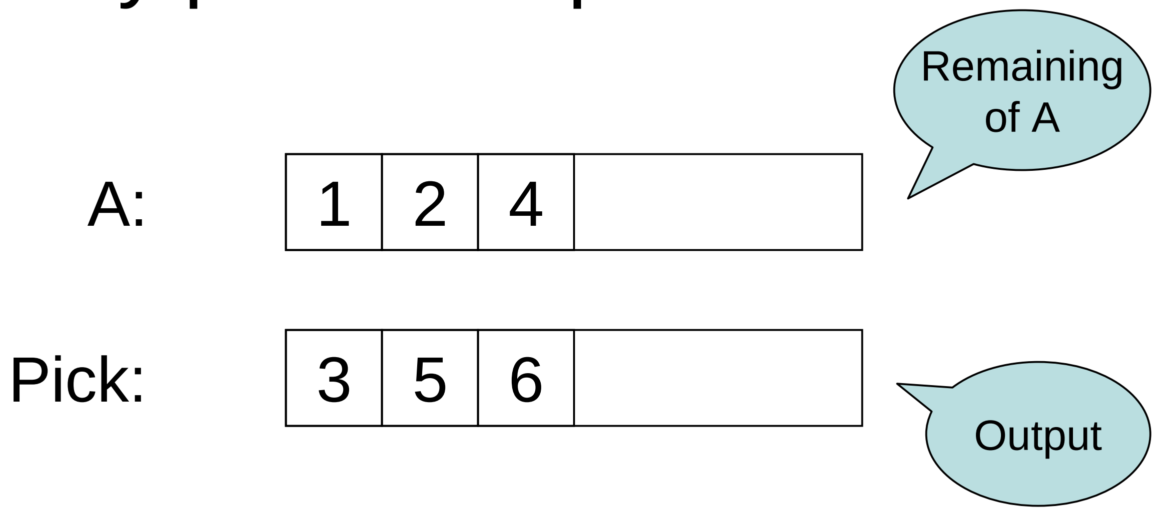
Question 5

- Greedy pick example:



Question 5

- Greedy pick example:



Question 5

- Greedy pick example:

Input:	3	1	2	5	6	4
Output:	3	5	6			

- Note: Not the longest increasing sequence 1, 2, 5, 6. And can not select 4 although it is after 3.

Question 5

- Sorting by greedy pick

A:

3	1	5	4	2

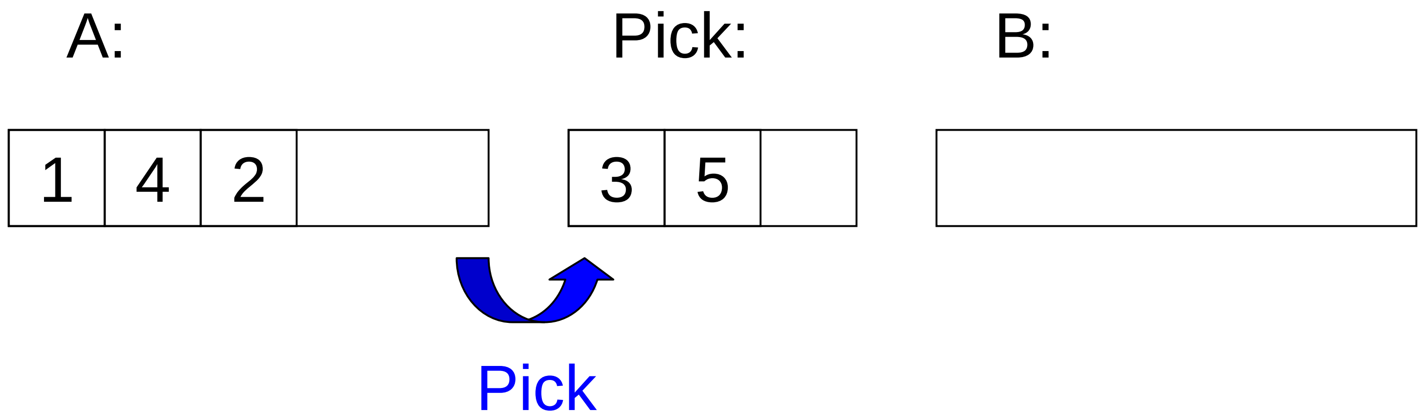
Pick:

B:

Intput

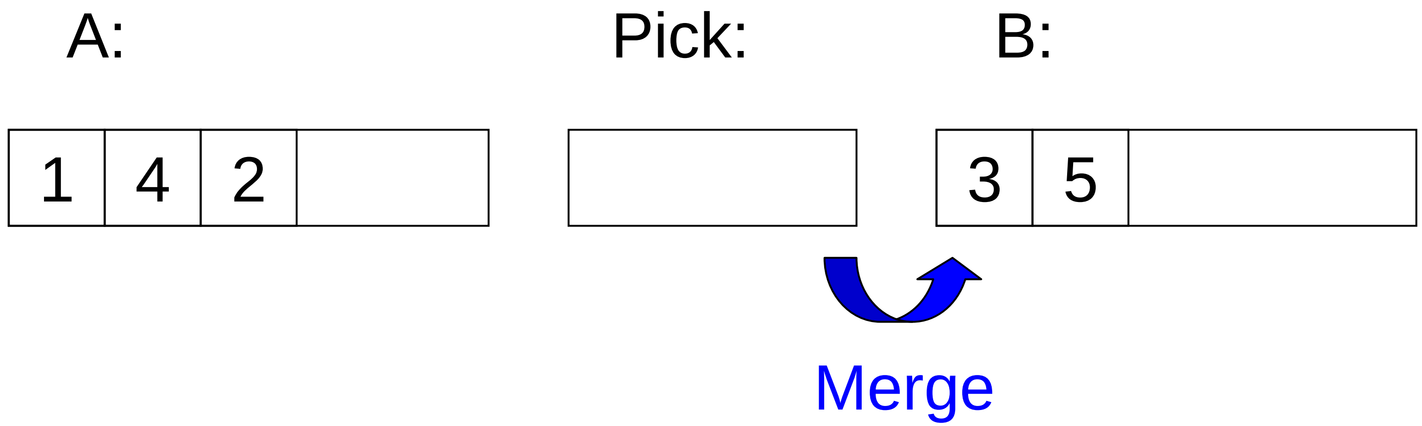
Question 5

- Sorting by greedy pick



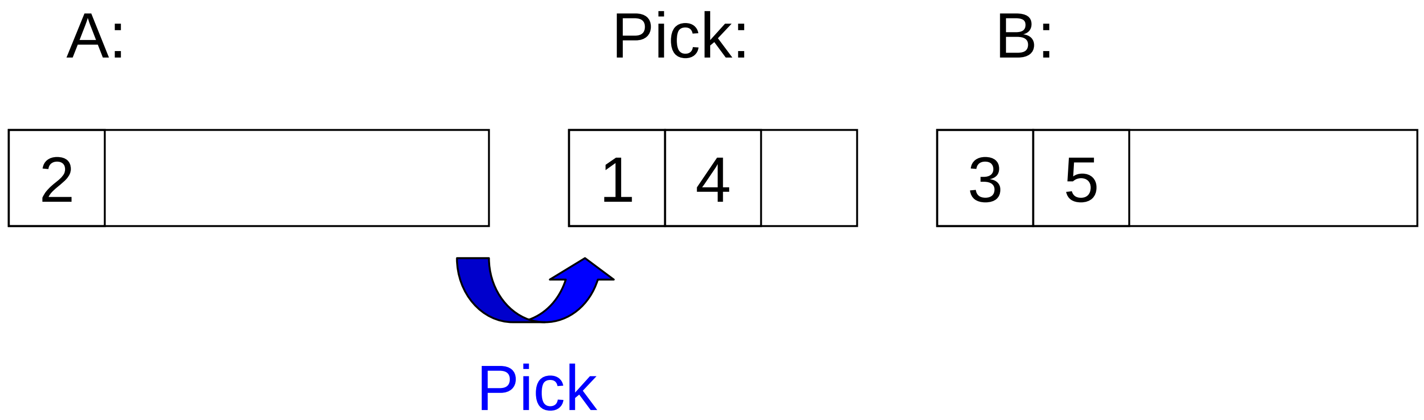
Question 5

- Sorting by greedy pick



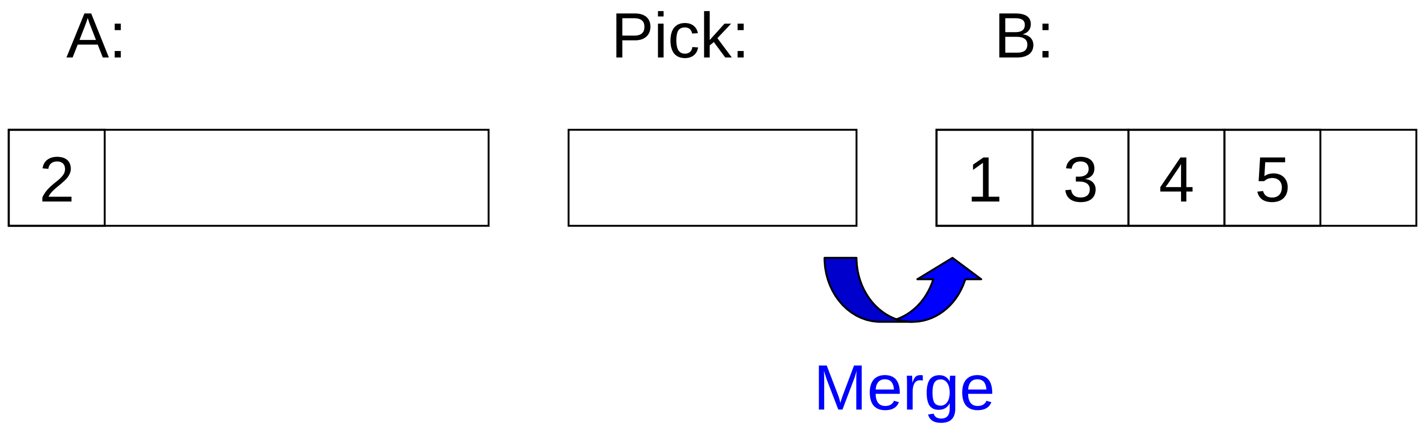
Question 5

- Sorting by greedy pick



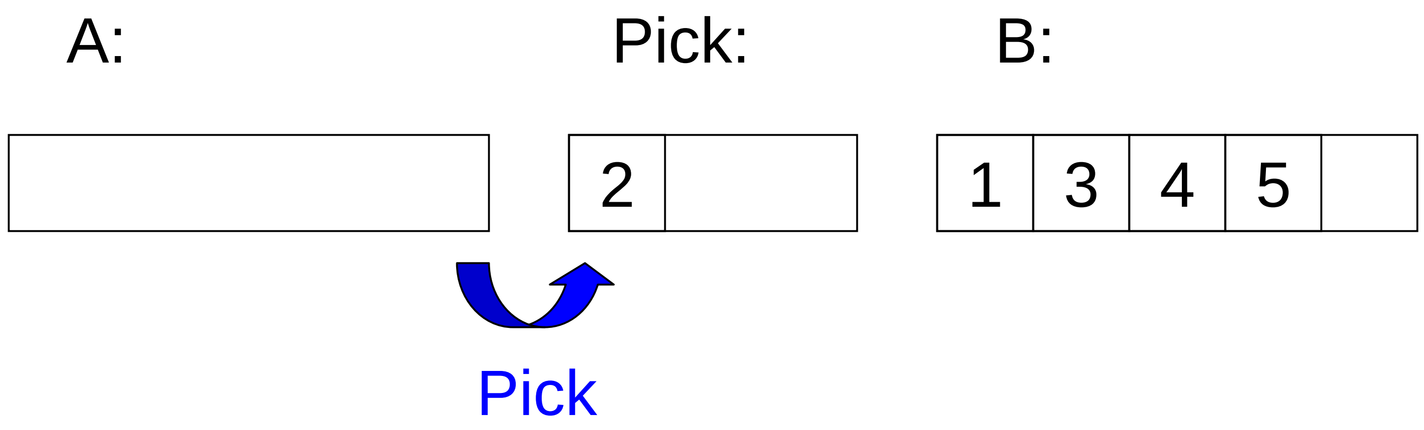
Question 5

- Sorting by greedy pick



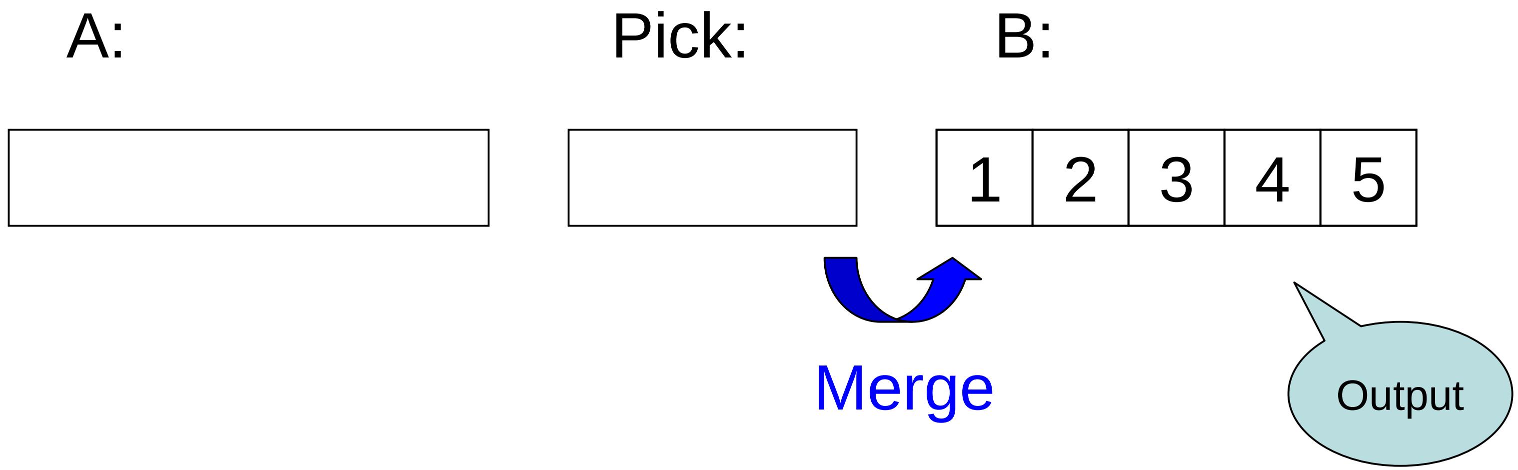
Question 5

- Sorting by greedy pick



Question 5

- Sorting by greedy pick



Question 5

- Show the sorting algorithm is correct
- Describe a worst-case input such that the above algorithm will run in $\Theta(n^2)$ time, and show your analysis

Question 6(a)

- Your friend is holding a sorted integer array A which contains n integer.
- The integers in A are from 0 to n and they are distinct
  - So there's an integer missing!
- Try to find the missing integer by asking $O(\log^2 n)$ questions in the form:

  "What is the jth bit of A[i]?"

Question 6(a)

- If we can ask

  "What is A[i]?"

  then we can find the number easily by binary search
- But now we can only see a bit at a time

- Note: n is the number of integers, not the number of bits
  - If we see all the bits, it's not $O(n)$

Question 6(b)

- Just like 6(a), if A is unsorted, can you find the missing integer in the running time of $O(n)$ questions?

- Hint: If you have seen the last bits of every A[i], can you determine if the missing number is odd or even?
- Hint: What is $n + n/2 + n/4 + n/8 + \ldots$?

Score

	Question 1	15%
	Question 2	15%
	Question 3	15%
	Question 4	15%
	Question 5	15%
	Question 6(a)	15%
+)	6(b)	15%
		105%
x)	Question 7	10%
		105% + 105% * 10%