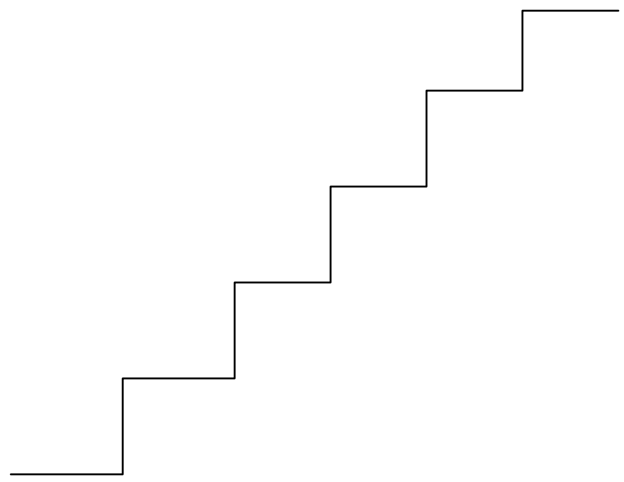
CS4311
Design and Analysis of Algorithms

Tutorial: Assignment 3 Hints

Question 1

- A stair has n steps
- Each time, Jack can walk up 1, 2, or 3 steps
- How many ways can Jack walk up the stair ?



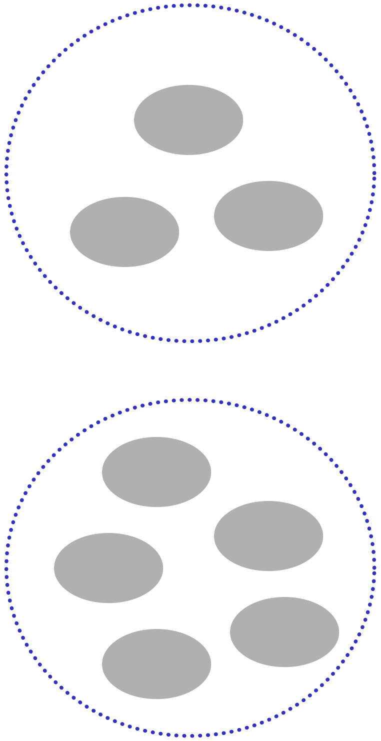
- Dynamic Programming

Question 2

- 2 piles of coins
- 2 players take turn to get coins
- Each turn, can either get

  (1) any #coins from one pile, or

  (2) same #coins from both piles
- Lose if does not get any coin in his turn



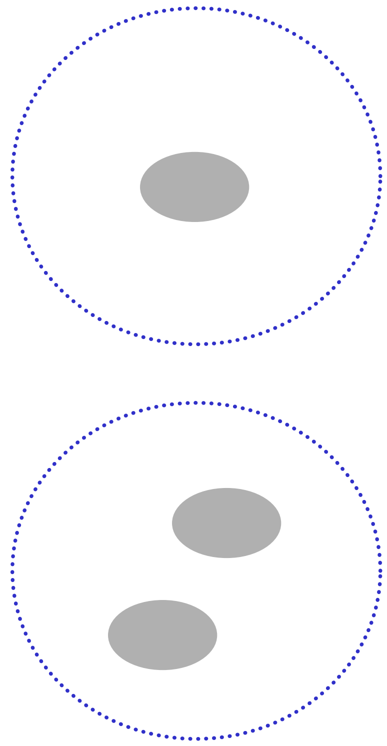
Question 2

- Assume both players are clever

➔ some combination are losing

E.g., If initially, the piles contain 1 coin and 2 coins, respectively, then we have only four ways of getting coin in this turn …

(What are they?)

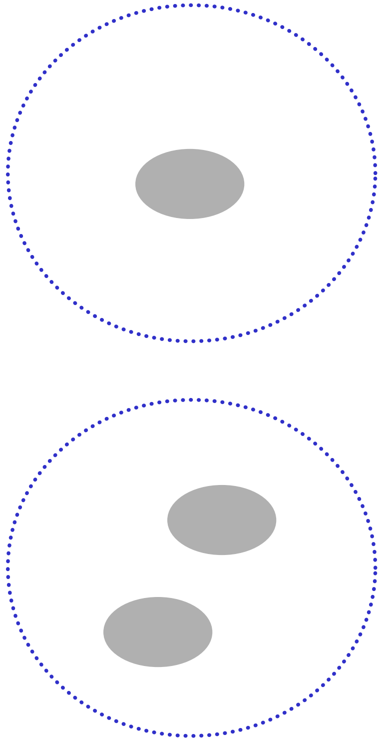


Question 2

Let (x,y) denote the status with x coins and y coins in the piles

- 4 ways to get coins from (1,2):
  - Case 1: (1,2) → (0,2)
  - Case 2: (1,2) → (1,1)
  - Case 3: (1,2) → (1,0)
  - Case 4: (1,2) → (0,1)

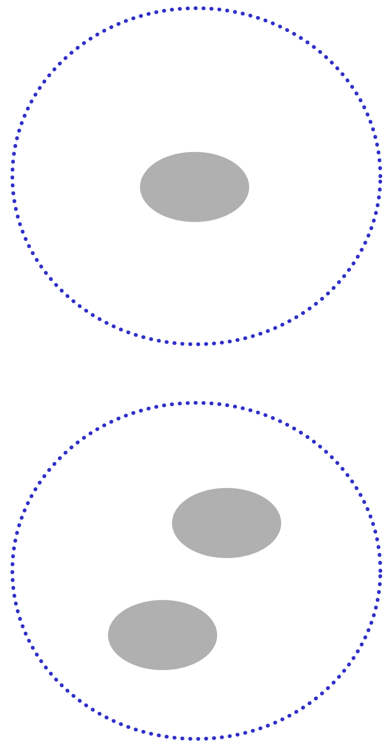
➔ All are losing ➔ (1,2) is losing



Question 2

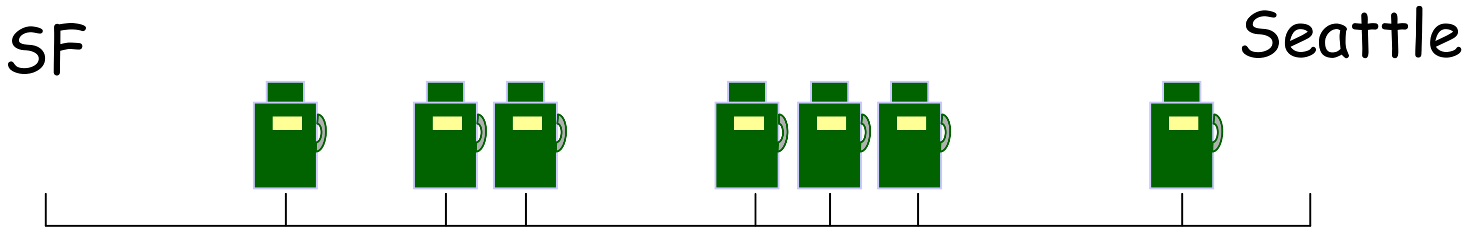
- In general, can we determine if (x,y) is a losing combination?

- Dynamic Programming



Question 3

- John has a car, which can travel n km when gas tank is full
- k gas stations between SF and Seattle
- How to stop for fewest # of stations ?



- Greedy Algorithm

Question 4

- If a Min-Heap contains n elements
  - Extract-Min takes O(log n) time
  - Insert takes O(log n) time
- Design potential function so that:
  - Extract-Min : O(1) amortized time
  - Insert : O(log n) amortized time

Question 4

- Extract-Min deletes some leaf node
- Each node should store some potential to prepare for its future deletion
- Potential function can be defined as the sum of the potentials in each node
- Question:

  What potential should we store in a node?

  [There are also other ways to assign the potential]

Question 5

- A sorted array supports fast searching, but slow insertion
- Can we trade searching time for insertion time ?

- Our Scheme:

  Partition n elements based on its binary representation of n

Question 5

- Let $k = \lceil \log (n+1) \rceil$, and

$$\langle b_{k-1}, b_{k-2}, \ldots, b_2, b_1, b_0 \rangle$$

  be binary representation of n

- Partition n elements to k sorted arrays such that:
  - if $b_j = 1$, array A_j holds 2^j elements
  - if $b_j = 0$, array A_j is empty

Question 5

For example,

- when n = 5, we have $5_{(dec)} = 101_{(bin)}$

  ➔ two non-empty arrays, one with 4 elements, one with 1 element

- when n = 11, we have $11_{(dec)} = 1011_{(bin)}$

  ➔ three non-empty arrays, with 8, 2, 1 elements, respectively

Question 5

- Searching time: $O(k \log n) = O(\log^2 n)$
- Insertion time:
  - Have to change the partitioning & to make sure each array is sorted
  - How to do so with amortized insertion time = $O(\log n)$?

- *Aggregate Method* should be the easiest among the three methods

Question 6 (Bonus)

- An extension to Question 2
- We show a method to generate losing combinations:

```
Set L_0 = (0,0)
for k = 1,2, ... {
   Set v = smallest unseen positive # ;
   Set L_k = (v, v+k) ;
}
```

Question 6 (Bonus)

$L_0 = (0,0)$

for k = 1,2, … {

  v = smallest unseen postive # ;

  $L_k = (v, v+k)$;

}

Interesting but unrelated fact:
This function generates each positive # exactly once

Sample Run:

$L_0 = (0,0)$

$L_1 = (1,2)$

$L_2 = (3,5)$

$L_3 = (4,7)$

$L_4 = (6,10)$

$L_5 = (8,13)$

…

Question 6 (Bonus)

- Part (a) and (b):
  - properties about losing combinations
- Part (c):
  - Only losing combinations are generated
- Part (d):
  - a losing combination containing x must be generated (in $O(x)$ steps)
- Part (e) and (f): running time = $O(n)$