

CS4311
Design and Analysis of
Algorithms

Classwork for Lecture 2

Exercise on Θ notation

Show the following bound:

1. $\sum_{k=1 \text{ to } n} k = 1+2+3+\dots+n = \Theta(n^2)$

Exercise on Θ notation

Show the following bound:

$$2. \quad \sum_{k=1 \text{ to } n} k^2 = 1+4+9+\dots+n^2 = \Theta(n^3)$$

Exercise on Θ notation

Show the following bound:

$$3. \sum_{k=1 \text{ to } n} (1/k) = 1 + (1/2) + \dots + (1/n) = \Theta(\log n)$$

More Exercises

4. For $|c| < 1$, can you simplify

$$\sum_{k=0 \text{ to } \infty} c^k = 1 + c + c^2 + \dots ?$$

More Exercises

5. For $|c| < 1$, can you show that

$$\begin{aligned}\sum_{k=1 \text{ to } \infty} kc^k &= c + 2c^2 + 3c^3 + \dots \\ &= c/(1-c)^2 ?\end{aligned}$$

More Exercises

6. Can you simplify this summation?

$$\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}$$

Challenge

What is the name of this sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, ... ?

- (a) Can you write a recursive program finding the n^{th} term? What will be the running time?
- (b) Can you write a better program to improve the running time to $O(n)$?
- (c) Can you think of an even faster way?