Design and Analysis of Algorithms

Assignment 2 Solution

Question 1

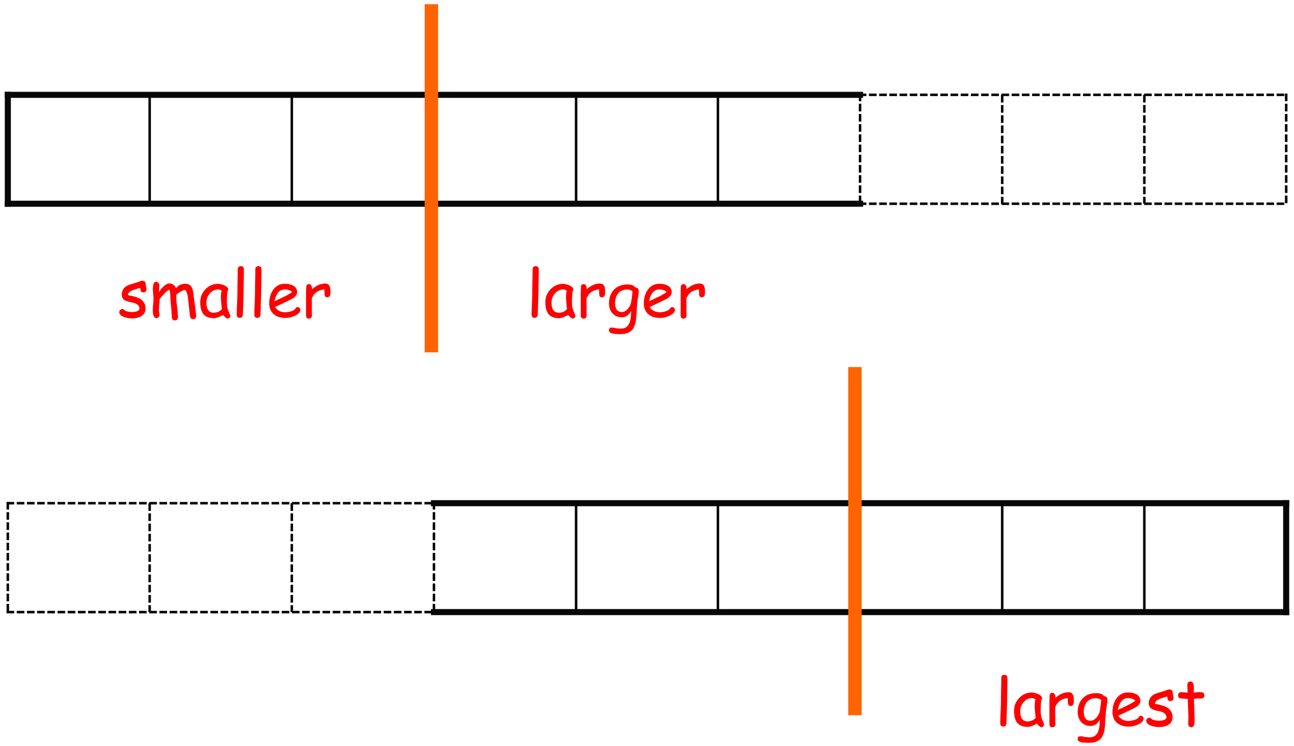
(a) Show by induction:

- Let l= the length of array, $m=\lceil l-3 \rceil$
- Show that an array of length l is sorted after running our algorithm
- For m=1:
  - l = 1, the element is sorted
  - l = 2, after swapping, this part is sorted
  - l = 3, this is similar to an insertion sort on array length of 3 (why?)

Question 1

- Assume m=n ,this algorithm is correct
- For m=n+1:
  - m%3= 1, we will put the smallest element on the first place and …
  - m%3= 2, just the same as m%3=1
  - m%3= 0, this is a little complicated…

- We have 3 routines sorting 2/3 total elements, by assumption, these routines correctly sort these elements.



Question 1

(b)

$$T(n) = 3T(2n/3) + O(n)$$

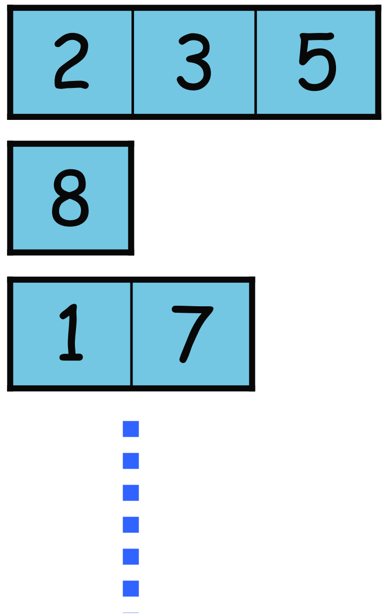
(c)

Master Theorem can easily give a tight bound

Question 2

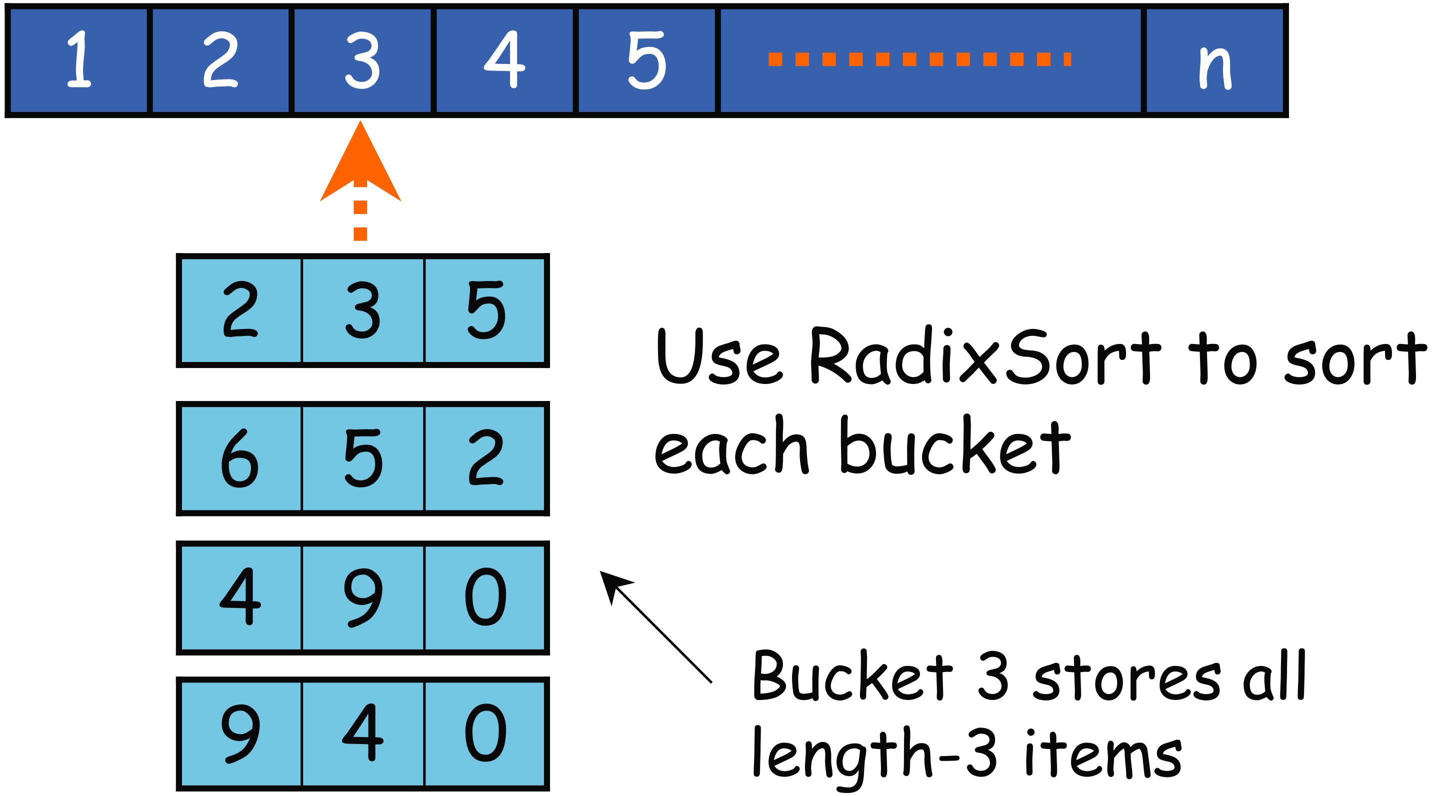
- An ordinary RadixSort can handle this.

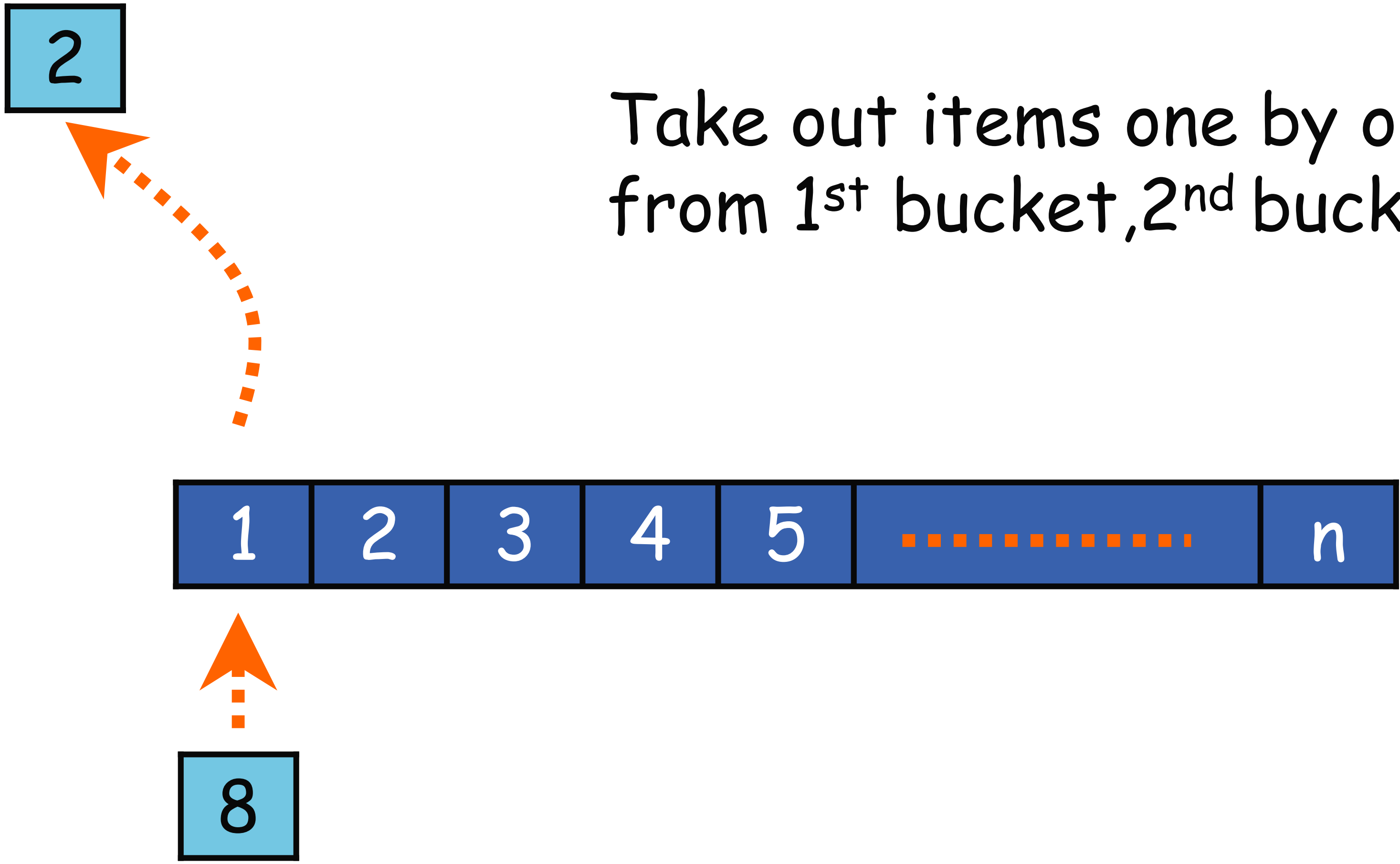
Question 3



"Bucketing by Length":

Put each item in the bucket that represents its length


1
2
3
4
5
n
2 3 5
6 5 2
4 9 0
9 4 0
Use RadixSort to sort each bucket
Bucket 3 stores all length-3 items



Take out items one by one from 1st bucket, 2nd bucket,...

Question 3

- Why is this algorithm correct?
- How to analyze the time complexity?

Question 4

(a) Easy , free points

(b) By the characteristic of Young Tableau, if Y[1,1] is infinity then no other elements can be greater than it, so …

Question 4

(c) Use a Heap-like method, maximum # of swapping steps = m+n-1

- To show correctness, use induction to show that, after each swap:
  - (I) at most one entry may violate Young Tableau property
  - (II) If an entry X violates the property, the entries above or left of X have values smaller than the entries down or right of X

Question 4

- (d)
  - First insert each unsorted elements into the tableau - $O(n^2(n+n)) = O(n^3)$
  - Then use Extract-Min n^2 times
    - $O(n^2(n+n)) = O(n^3)$
  - Total : - $O(n^3) + O(n^3) = O(n^3)$

Question 5 (Bonus)

- Let $Bound_i(K)$ = position of the boundary on i^{th} row which pivots K

1	2	4	6
3	5	7	8
6	8	9	12
9	10	11	13

$Bound_1(5)=3$

Question 5

- Observe that for any K,

  $\mathrm{Bound}_i(K) \geqq \mathrm{Bound}_{i+1}(K)$ (Why?)

1	2	4	6
3	5	7	8
6	8	9	12
9	10	11	13

Question 5

- Then we search for the element just before the boundary

1	2	4	6
3	5	7	8
6	8	9	12
9	10	11	13

Question 5

- How about its complexity?
- Suppose there're n rows, m columns
- Finding $Bound_1(K)$ – $O(n)$
- Search for $Bound_i(K)$ from tail – $O(n+m)$
- Find K – $O(m)$
- Total : $O(n) + O(n) + O(m) = O(n+m)$