

# User Selection and Decoding Precedence Based on the Anisotropic Orthogonal Procrustes Analysis for Uplink Multi-User MIMO

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**Abstract**—The advent of multi-user multiple input multiple output (*MU-MIMO*) technology enables concurrent transmissions between the access point and user stations. In this paper, we address MU-MIMO with zero-forcing successive interference cancellation (*ZF-SIC*) decoder in the uplink direction. To maximize the throughput gain while reducing the computational complexity, we propose a novel algorithm consisting of two parts—user selection and decoding precedence. The user selection part exploits the anisotropic orthogonal Procrustes analysis (*AOPA*) and the decoding precedence part is based on signal-to-noise ratio (*SNR*). Simulation results show that our algorithm is fast and outperforms existing schemes in terms of sum rate.

**Keywords**—uplink MU-MIMO, antenna selection, interference cancellation

## I. INTRODUCTION

The increasing demand for various applications such as 4K video, driverless vehicles, and Internet of Things have been leading to the research and development of high-rate wireless access. An important technique to achieve such a rate demand is multiple input multiple output (*MIMO*), which offers significant gains in data rate and link reliability without consuming extra bandwidth. For the scenario with multiple users, multi-user MIMO (*MU-MIMO*) is beneficial because interference from multiple users can be nullified or well managed by MU-MIMO technology.

In a WiFi environment, an access point (*AP*) is typically equipped with a multitude of antennas whereas a user station (or called a *user*) often has only one antenna due to its form factor. (The mismatch also happens in contemporary and next-generation mobile networks where a base station typically has more antennas than a mobile station has.) In such a situation, it is beneficial for the multi-antenna AP to simultaneously serve multiple single-antenna users in the uplink direction. Indeed, MU-MIMO enables a  $N_r$ -antenna AP to concurrently receive  $\min(N_t, N_r)$  data streams sent from  $N_t$  single-antenna users. So MU-MIMO may achieve a throughput gain of up to  $N_r$  times, compared with the single-user case.

To fully utilize the available degrees of freedom [8], recent works such as [1]-[10] recommended developing MU-MIMO wireless local area networks (*WLANs*) to enable concurrent transmissions from multiple users, with emphasis on issues

such as medium access control [1], [2], user/antenna selection [4], [10], and practical implementation [5], [6], [9], [10].

Zhou and Niu proposed in [1] an uplink medium access protocol for multiple-antenna WLANs [1], which is based on IEEE 802.11 Distributed Coordination Function. The whole transmission procedure is divided into two random access period and data transmission period. Users compete for the channel by RTS/CTS exchanges in random access period and the winners can transmit data to the AP simultaneously in data transmission period. To reduce the overhead in exchanging control messages, Tandia et al. [2] designed a low-overhead protocol which exchanges control messages in the OFDMA manner.

From the user selection perspective, Zhong and Xu [4] considered joint user and transmit antenna selection for uplink MU-MIMO systems. To enhance sum rate, they developed two fast algorithms—the capacity-based selection algorithm (*CB-JUTAS*) algorithm and the norm-based selection algorithm (*NB-JUTAS*). *CB-JUTAS* which takes inter-user interference into account decouples user selection and antenna selection into two independent sub-procedures, whereas *NB-JUTAS* is an egoistic algorithm in the sense that it ignores inter-user interference. Unlike *CB-JUTAS* and *NB-JUTAS*, *MIMOMate* proposed in [10] selects users according to channel orthogonality, while ensuring all users fairly share concurrent transmission opportunities.

There is much room left for improving the user selection algorithms aforementioned and major reasons are given as follows: *CB-JUTAS* and *NB-JUTAS* do not consider the real constraint that a practical wireless network only supports a finite set of modulation and coding schemes (*MCSs*). *MIMOMate* encounters performance loss because the leader-follower relationship determined a priori together with the constraints 3 and 4 in [10] make *MIMOMate* ignore a large portion of the entire feasible set. To maximize throughput gain brought by uplink MU-MIMO, it is crucial to select a subset of users to send packets concurrently and to determine an appropriate decoding precedence for the selected users, which motivates us to propose a novel algorithm in this paper.

Our algorithm consists of two parts—user selection and decoding precedence. Unlike the aforementioned algorithms, the user selection part of our algorithm, which aims to choose the users with high signal-to-noise ratio (*SNR*) and good

channel orthogonality, exploits the anisotropic orthogonal Procrustes analysis (AOPA). The decoding precedence part of our algorithm is based on SNR of the  $N_r$  users selected in the first part. The second part has a worst-case complexity of  $O(m \log m)$ , where  $m$  is the number of supported MCSs.

The remainder of this paper is organized as follows. Section II describes the system model used in this paper. Section III presents AOPA-based user selection, which is the first half of our proposed algorithm; while the second half, SNR-based decoding precedence, is introduced in Section IV. The performance evaluation of our algorithm is presented in Section V. And Section VI gives some concluding remarks.

## II. SYSTEM MODEL AND PRELIMINARY

Same as [2] and [10], we consider the *uplink* MU-MIMO scenario in which there are  $N_t$  single-antenna users and one  $N_r$ -antenna AP. Each user has packets waiting to be sent and thus is a potential transmitter, whereas the AP is the receiver. We focus on the case when  $N_t > N_r$ ; in this case, the degree of freedom is  $\min(N_t, N_r) = N_r$ . Based on the channel state information (CSI), the AP needs to select  $N_r$  users to be transmitters and decide the decoding precedence of these  $N_r$  transmitters.

We confine this paper to open-loop MU-MIMO and zero-forcing successive interference cancellation (ZF-SIC) decoder. We focus on open-loop MU-MIMO because only the receiver/AP requires of knowing the CSI. (Otherwise obtaining CSI at the transmitters consumes a substantial amount of feedback bandwidth.) The CSI is specified by the  $N_t \times N_r$  channel matrix

$$H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{N_t} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,N_r} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,N_r} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_t,1} & h_{N_t,2} & \cdots & h_{N_t,N_r} \end{bmatrix}$$

where  $H_n$  is the  $n^{\text{th}}$  row of  $H$  and  $h_{n,m}$  is the channel gain from the  $n^{\text{th}}$  user to the  $m^{\text{th}}$  antenna at the AP. The AP also knows  $\rho_n$ ,  $1 \leq n \leq N_t$ , which is the ratio between the transmission power of user  $n$  and the noise power  $\sigma^2$ . Since the AP knows CSI and  $\rho_n$ , the AP can compute the square root of SNR matrix (*sqrtsNR matrix*)

$$G = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_{N_t} \end{bmatrix} = \begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,N_r} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,N_r} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N_t,1} & g_{N_t,2} & \cdots & g_{N_t,N_r} \end{bmatrix}$$

where  $G_n$  is the  $n$ -th row of  $G$  and  $g_{n,m} = h_{n,m} \sqrt{\rho_n}$ . We name  $G_n$  as the *sqrtsNR vector* of user  $n$ .

After  $N_r$  users have been selected for concurrent transmissions, ZF-SIC is assumed to be used to decode these users' packets because it can achieve high spectral efficiency with reasonable decoding complexity. We explain the ZF-SIC decoder by an example. Consider the case of  $N_t = N_r = 2$ . As shown in Fig. 1, users 1 and 2 send  $s_1$  and  $s_2$ , respectively. As shown by the green arrow in Fig. 1, the *receive vector* is

$R = [r_1 \ r_2] = \sigma G_1 s_1 + \sigma G_2 s_2 + N$ , where  $N = [n_1 \ n_2]$  is the noise vector.  $n_1$  and  $n_2$  are independent and follow the complex Gaussian distribution,  $n_1, n_2 \sim \text{CN}(0, \sigma^2)$ .  $\sigma^2$  is the average noise power.

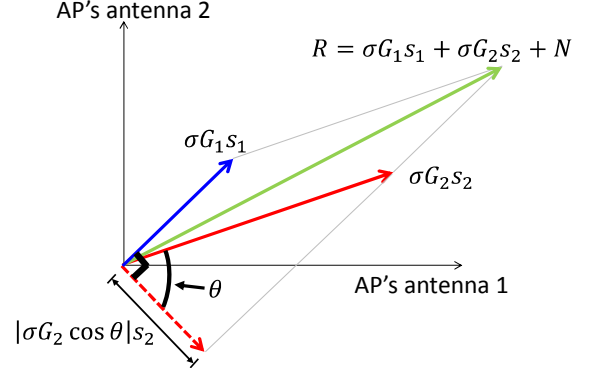


Fig. 1. Illustration of ZF-SIC in the case of  $N_t = N_r = 2$ .

Initially, the *residual vector* is the receive vector  $R$ . To extract any of the concurrent packets, say  $s_2$ , the AP nullifies the interference from other packets by projecting the residual vector onto the null space of all  $G_i$ s except  $G_2$  (corresponding to the packets not decoded yet); this projection is called the *effective projection* of user 2. In this example, the AP projects  $R$  on the direction orthogonal to  $G_1$ . So user 2's effective projection is the red dashed arrow in Fig. 1. After projection, the AP obtains  $s_2$  with an effective SNR value [6] decreased by a factor of  $\cos^2 \theta$ :

$$\text{SNR}_{\text{effective}} = \text{SNR}_{\text{original}} \cos^2 \theta \quad (1)$$

where  $\theta$  is the angle between  $G_2$  and the null space.

By the Shannon-Hartley theorem,  $s_2$  cannot be transmitted at a rate exceeding  $\log_2(1 + \text{SNR}_{\text{effective}})$  bps/Hz. In practice, the transmission rate is further restricted by a finite set of supported MCSs. Table 1 lists the supported MCSs [12], data rates, and corresponding SNR thresholds [13] in IEEE 802.11ac standard. Once the effective SNR value is computed by (1), the transmission rate is the highest data rate whose SNR threshold is below the effective SNR.

TABLE 1. Selected 802.11ac rates (short guard interval)

MCS	Modulations & ratio	Data rate (Mbps)	Required SNR
0	BPSK 1/2	15	2 dB $\approx$ 1.6
1	QPSK 1/2	30	5 dB $\approx$ 3.2
2	QPSK 3/4	45	9 dB $\approx$ 7.9
3	16-QAM 1/2	60	11 dB $\approx$ 12.6
4	16-QAM 3/4	90	15 dB $\approx$ 31.6
5	64-QAM 2/3	120	18 dB $\approx$ 63.1
6	64-QAM 3/4	135	20 dB $\approx$ 100
7	64-QAM 5/6	150	25 dB $\approx$ 316
8	256-QAM 3/4	180	29 dB $\approx$ 794
9	256-QAM 5/6	200	31 dB $\approx$ 1258

Knowing  $s_2$ , the AP can easily reconstruct  $\sigma G_2 s_2$ , which is called the *component* from user 2 (or user 2's component) and is drawn by the red solid arrow in Fig. 1. The next step is to update the residual signal by subtracting the reconstructed component  $\sigma G_2 s_2$ . After that, the AP can extract any of the

remaining packets, one at a time, by repeating the procedure aforementioned.

Note that the sum rate is affected not only by the users that are selected for concurrent transmissions but also by the order in which these selected users' packets are decoded. To maximize the sum rate, we observe three rules of thumb:

- First, it is preferred to select the users with good channel orthogonality. That is, the components of selected users should be as mutually orthogonal as possible, which can reduce the SNR loss during projection.
- Second, the SNR value of selected users should be high. Equivalently, it is preferred to select the users whose components are among the lengthiest.
- Third, it would be great if selected users retain the same rates before and after ZF-SIC's projection on the null space. In other words, selected users had better have enough margins to tolerate the SNR loss incurred by projection.

The first half of our algorithm, AOPA-based user selection, considers the first two rules; while the second half of our algorithm, SNR-based decoding precedence, takes the third rule into consideration.

### III. AOPA-BASED USER SELECTION

This section introduces the first half of our algorithm, AOPA-based user selection. As mentioned in Section II, among  $N_t > N_r$  users, the AP should select  $N_r$  users whose components are as orthogonal and long as possible. To this end, we exploit the anisotropic orthogonal Procrustes analysis (AOPA) [11]. What follows explains how the AOPA-based user selection method we propose works with the help of the virtual antenna technique we devise.

Let us consider a degenerated problem before delving into the actual user selection. Pretend that there are exactly  $N_r$  users whose components are of unit length and pretend that our goal is the orthogonality test—testing how orthogonal these  $N_r$  components are. To test the orthogonality, we can simply use the orthogonal Procrustes analysis:

$$\arg \min_Q \|I_{N_r} - GQ\| \quad (2)$$

subject to the constraint that  $Q$  is a unitary (square) matrix. The notation  $\|\cdot\|$  means the Frobenius norm.  $I_{N_r}$  is the  $N_r$ -by- $N_r$  identity matrix whose rows are the standard basis for the  $N_r$ -dimensional Euclidean space. Any square matrix right-multiplied by a unitary matrix can be regarded as rotating the rows of the square matrix. Because  $\|A - B\|$  quantifies the difference between any two matrices  $A$  and  $B$ , (2) is essentially equivalently to find out how well the  $N_r$  unit-length components (i.e., the rows of  $G$ ) approximates to the standard basis (i.e., rows of  $I_{N_r}$ ) after rotating by any angle.

What if our goal is still the orthogonality test but now the  $N_r$  components are no longer of unit length? In this situation, we can simply use the anisotropic orthogonal Procrustes analysis (AOPA):

$$\arg \min_{\Lambda, Q} \|I_{N_r} \Lambda - GQ\| \quad (3)$$

subject to the constraints that  $Q$  is a unitary (square) matrix and  $\Lambda$  is a diagonal (square) matrix. Because  $I_{N_r}$  multiplied by a diagonal matrix means scaling in the directions of the standard basis, (3) is essentially equivalently to find out how well the  $N_r$  components (i.e., the rows of  $G$ ) approximates to the standard basis (i.e., the rows of  $I_{N_r}$ ) after scaling by any factor and rotating by any angle.

Now let us go back to our real goal—selecting  $N_r$  users from  $N_t$  users such that the components of selected users are as orthogonal and long as possible. In general, AOPA has the form

$$\arg \min_{\Lambda, Q} \|X\Lambda - YQ\| \quad (4)$$

and is subject to the constraints that  $Q$  is a unitary (square) matrix and  $\Lambda$  is a diagonal (square) matrix

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}$$

where  $\lambda_i$ s are the scaling factors. We regard  $\lambda_i$  as an estimate of the length of effective projection of user  $i$ .

Because  $G$  (whose dimension is  $N_t \times N_r$ ) is not a square matrix, we cannot apply AOPA directly due to dimension mismatch. To solve this dimension mismatch problem, we devise the technique called *virtual antenna*.

What follows explains the virtual antenna technique. Although the AP has only  $N_r$  antennas, we pretend that additional  $N_t - N_r$  virtual antennas is added to the AP. The only purpose of adding these virtual antennas is to match the dimensions of matrices in the AOPA. These virtual antennas themselves are imaginary; therefore, the channel gains corresponding to the virtual antennas are zero. After adding virtual antennas, we set  $Y$  to be the extended version of the sqrtSNR matrix:

$$Y = \underbrace{\begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,N_r} & \overbrace{0 \ 0 \ \cdots \ 0}^{N_t - N_r} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,N_r} & 0 \ 0 \ \cdots \ 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{N_t,1} & g_{N_t,2} & \cdots & g_{N_t,N_r} & 0 \ 0 \ \cdots \ 0 \end{bmatrix}}_{N_t}$$

and  $X$  is set to be the  $N_r$ -by- $N_t$  identity matrix whose rows are the standard basis for the  $N_r$ -dimensional Euclidean space:

$$X = I_{N_r}$$

With  $X$  and  $Y$  defined above, all the dimensions of the matrices in (4) are  $N_t \times N_t$ . In this way, we have solved the dimension mismatch problem, which enables us to apply AOPA in our user selection method.

Now it is time to find out the optimal  $Q$  and  $\Lambda$  in (4), given  $X$  and  $Y$ . We define the matrix  $B$  as  $B = Y'X$ , where  $Y'$  is the transpose of  $Y$ . To solve the AOPA problem in (4), we use the block relaxation (BR) algorithm [11] which is an iterative algorithm consisting of the following two stages:

- Given  $Q$ , the global optimum  $\Lambda$  is attained when  $\Lambda$  has the following diagonal elements:

$$\lambda_i^* = \frac{(B'Q)_{i,i}}{(X'X)_{i,i}}, \quad i=1,2,\dots,N_t$$

- Given  $\Lambda$ , the global optimum  $Q$  is computed by the orthogonal polar factor of the matrix  $B\Lambda$ . That is, after from the singular value decomposition  $B\Lambda = USV'$ , we obtain  $Q = UV'$ .

The BR algorithm is run iteratively until the stopping condition is satisfied. Once BR completes, the AP selects top  $N_r$  diagonal elements in  $\Lambda$ ; the corresponding  $N_r$  users are selected for concurrent transmissions. For example, if  $\lambda_1^* > \lambda_2^* > \dots > \lambda_{N_r}^* > \lambda_{N_r+1}^* > \dots > \lambda_{N_t}^*$ , then users 1, 2, ...,  $N_r$  are selected and granted for concurrent transmissions.

For acceleration purpose, the stopping condition in our simulation is that the users corresponding to top  $N_r$  scaling factors (i.e.,  $\lambda_i$ s) keeps the same for three consecutive iterations. The reason behind is that we aim to choose  $N_r$  users whose effective projections are among the top  $N_r$  long. The values of  $\lambda_i$ 's are important because they are approximations of the length of effective projections. On the contrary, the exact values of  $Q^*$  and  $\Lambda^*$  do not really matter in our user selection method.

#### IV. SNR-BASED DECODING PRECEDENCE

This section introduces the second half of our algorithm, SNR-based decoding precedence. After selecting  $N_r$  users out of  $N_t$  users, the AP needs to decide the order in which the ZF-SIC decoder extracts the packets sent by the  $N_r$  users. This precedence affects effective SNR: For the user whose packet is decoded last, its effective SNR is equal to its (original) SNR and thus this user encounters no SNR loss during ZF-SIC. On the contrary, the first decoded packet has a high probability to suffer from a significant SNR loss caused by the projection.

In reality, the effective SNR determines the transmission rate in a discrete manner. As mentioned in Section II, only a finite set of MCSs are supported in practice. To achieve a high sum rate with low complexity, we develop a simple yet efficient method that determines decoding precedence based on (original) SNR instead of effective SNR.

The decoding precedence method we propose has three rules. Rule 1 tells that an early decoded user should have a large *SNR loss margin*<sup>1</sup> to tolerate the possibly large SNR loss during ZF-SIC (since an earlier decoded user has a higher chance to encounter a significant SNR loss); on the contrary, a user with a small SNR loss margin should be decoded latter.

To implement Rule 1, each supported MCS is categorized into two classes—the lower and upper classes—based on the (original) SNR value. Any SNR value lying in a lower class can be regarded as having a small SNR loss margin, whereas each upper class implies a large SNR loss margin. Precedence values are assigned to all of these classes, as shown in Fig. 2

which uses the MCSs supported by IEEE 802.11ac and SNR thresholds given in Table 1 as an example. Because a smaller precedence value means earlier decoding, each lower class is assigned a precedence value that is larger than the precedence value of any upper class.

What if the SNR values of two users lie to the lower (upper) classes of two different MCSs? Rule 2 tells that in this situation, the user whose SNR corresponds to a higher-rate MCS should be decoded latter in order to maximize the sum rate. Therefore, as shown in Fig. 2, the precedence value of the lower (upper) class of MCS 0 is smaller than that of MCS 1, which is smaller than that of MCS 2, and so on so forth.

What if two users lie to exactly the same class of the same MCS? Rule 3 tells that in this situation, the user whose SNR is larger should be decoded latter to maximize the sum rate.

In summary, what our decoding precedence method does is simply as follows. Given the selected users and their SNR values, our decoding precedence method finds out the precedence values of these users by table lookup with the help of Fig. 2. The decoding precedence is basically the ascending order of the precedence values of these users. If there are two or more users whose precedence values are identical, their relative order is determined by their SNR—a higher SNR value means being decoded latter.

The decoding precedence method we propose is essentially a two-layer sorting algorithm. The first layer is the bucket sort with  $2m$  buckets, where  $m$  is the number of supported MCSs. The second layer is an ordinary sort (such as merge sort). So the complexity of our decoding precedence method is usually linear  $O(m + N_t)$ ; in the worst-case, the complexity is  $O(m + N_t + N_t \log N_t) = O(m + N_t \log N_t)$ .

Precedence value	11	12	13	14	15	16	17	18	19	20
	lower	lower	lower	lower	lower	lower	lower	lower	lower	lower
	upper	upper	upper	upper	upper	upper	upper	upper	upper	upper
	MCS 0	MCS 1	MCS 2	MCS 3	MCS 4	MCS 5	MCS 6	MCS 7	MCS 8	MCS 9
SNR value	1.6	3.2	7.9	12.6	31.6	63.1	100	316	794	1258
	2.4	5.6	10.3	22.1	47.4	81.5	208	555	1026	1888

Fig. 2. A smaller precedence value means being decoded earlier. The supported MCSs and SNR thresholds are copied from Table 1.

#### V. PERFORMANCE EVALUATION

We evaluate our proposed algorithm under various situations and compare its performance with several existing algorithms by an in-house simulator written in MATLAB. All results are averaged over 500 instances.

The simulation setting is as follows. There are one AP and  $N_t$  user stations in the wireless network. Each user station has one transmit antenna; while the number of receive antennas at the AP is  $N_r = 2, 4, \text{ or } 8$ . The AP is located in the center of a circle with a radius  $R = 100$  meters, and the user stations are distributed uniformly over the circle.

It is assumed that signal propagation over wireless channels encounters path loss and Rayleigh-distributed multipath fading. The path loss exponent is denoted by  $\alpha$ . The channel gains  $h_{n,m}$ ,  $1 \leq n \leq N_t$  and  $1 \leq m \leq N_r$ , are independent complex Gaussian random variables with zero mean and unit variance. The channel is assumed to remain fixed during a period of

<sup>1</sup> The SNR loss margin of user  $i$  is defined as the maximum SNR loss without making user  $i$  falling to a lower-rate MCS. It is computed by  $|G_i|^2 / \eta_i$ , where  $\eta_i$  is the SNR threshold of the highest-rate MCS user  $i$  can use provided that no other user exists.

time (longer than 100 ms [7]), and the transmission power of every transmit antenna is set to be fixed and identical in the simulation. The sqrtSNR vector of user  $n$  can be expressed as

$$G_n = \sqrt{\text{SNR}_0 \left(\frac{r_n}{R}\right)^{-\alpha}} \cdot H_n \quad (5)$$

where  $\text{SNR}_0$  is the SNR value at the reference point that is located  $R$  meters away from the AP (provided that no other user exists) and  $r_n$  is the distance between user  $n$  and the AP. Some parameters used in the simulation are shown in Table 2.

TABLE 2. Simulation parameters

Parameter	Value
$R$	100 m
Path loss exponent, $\alpha$	3.7
$\text{SNR}_0$	10 dB

We compare the performance of our proposed algorithm with the algorithms—CB-JUTAS [4], NB-JUTAS [4], and MIMOMate<sup>2</sup> [10]. In addition, we also compare our algorithm with the exhaustive algorithm, which first uses the method mentioned in Section III to select  $N_r$  users and then searches all possible decoding orders exhaustively to get the optimal one. The goal of comparing our method with the exhaustive algorithm is to know how much room for further improvement is left for our decoding precedence method.

Important performance metrics include sum rate and runtime. Given the result of user selection and decoding precedence, the sum rate is computed by summing up the individual rates of the selected users. The individual rate of a selected user is computed by running the ZF-SIC decoder, getting the effective SNR of that user, and looking up Table 1 to map the effective SNR value to the individual rate.

The sum rates the five algorithms achieve are shown in Fig. 3, Fig. 4, and Fig. 5 when the AP is equipped with 2, 4 and 8 receive antennas, respectively. It is observed that in terms of sum rate, our proposed algorithm significantly outperforms other algorithms except the exhaustive algorithm. The outperformance over MIMOMate is because MIMOMate ignores a portion of the entire feasible set due to fairness constraints imposed. The outperformance of our algorithm over CB-JUTAS and NB-JUTAS is because CB-JUTAS and NB-JUTAS are not tailored to ZF-SIC decoder and do not consider the real constraint that a practical wireless network only supports a finite set of MCSs. Note that the performance gap (in sum rate) between our algorithm and the exhaustive algorithm is less than 5%. This small gap confirms that our algorithm is efficient.

Besides the sum rate results shown above, the runtime each algorithm takes is presented in Fig. 6, Fig. 7, and Fig. 8 when the AP is equipped with 2, 4 and 8 receive antennas, respectively. The runtime results show that our algorithm can complete in a short amount of time. Together with the sum rate results aforementioned confirms that our algorithm is fast and efficient. Note that the runtime results are obtained from

executing Matlab code; in real systems implementing in C/C++ or with hardware acceleration, our algorithm can complete in a much shorter time than what these figures show.

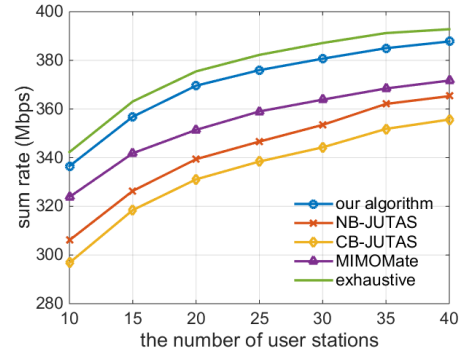


Fig. 3. The sum rate of each algorithm when  $N_r = 2$ .

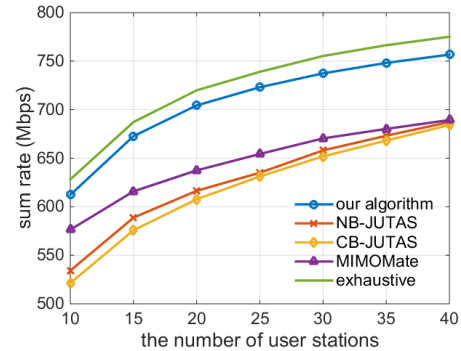


Fig. 4. The sum rate of each algorithm when  $N_r = 4$ .

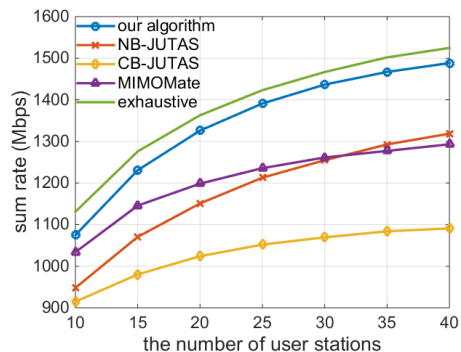


Fig. 5. The sum rate of each algorithm when  $N_r = 8$ .

From Fig. 6, Fig. 7, and Fig. 8, we observe that either MIMOMate or the exhaustive algorithm takes the longest time. The runtime of MIMOMate is long and its slope with respect to  $N_r$  is largest because *i)* MIMOMate uses a heuristic matching algorithm with a complexity of  $O(N_r N_r^2)$  which results in a number of  $N_r$ -dimensional matchings, and *ii)* extra time is needed to find out which one is the best  $N_r$ -dimensional matching. As to the exhaustive algorithm, its runtime becomes very long as  $N_r$  becomes large. This is because the exhaustive algorithm needs to evaluate all of  $N_r!$  permutations to find the optimal one.

<sup>2</sup> The MIMOMate algorithm gives a number of candidates for user selection and decoding precedence. Each simulation result of MIMOMate shown in the figures of this paper corresponds to the candidate with the highest sum rate.

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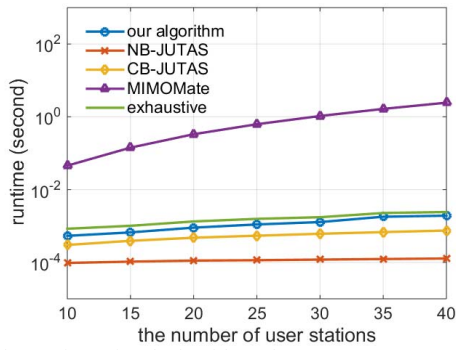


Fig. 6. The runtime of each algorithm when  $N_r = 2$ .

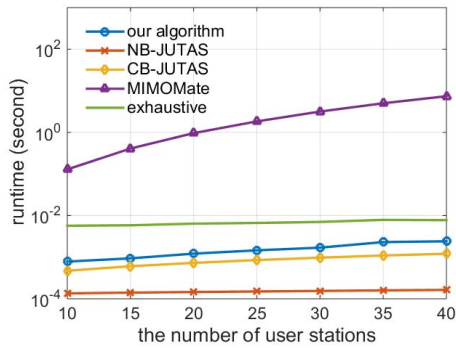


Fig. 7. The runtime of each algorithm when  $N_r = 4$ .

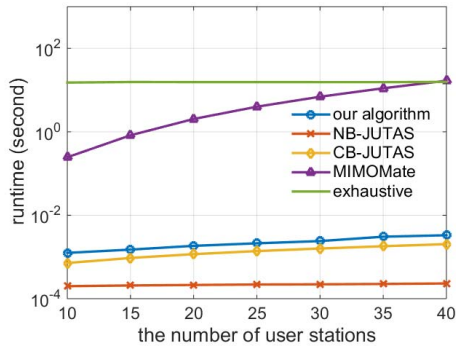


Fig. 8. The runtime of each algorithm when  $N_r = 8$ .

## VI. CONCLUSION

In this paper, we have addressed uplink MU-MIMO with ZF-SIC decoder at the receiver. We have proposed a novel algorithm which consists of two parts—user selection and decoding precedence. The user selection part exploits the anisotropic orthogonal Procrustes analysis with the help of the virtual antenna technique we devise. The decoding precedence part is based on SNR and has a worst-case complexity of  $O(m \log m)$ , where  $m$  is the number of MCSs supported in the system. Extensive simulation results show that compared to existing algorithms, the algorithm we propose is fast and has superior performance in terms of sum rate.