Efficient Network Coding at Relay for Relay-Assisted Network-Coding ARQ Protocols

Jung-Chun Kao and Kuo-Hao Ho
Department of Computer Science, National Tsing Hua University
Hsinchu, Taiwan

Abstract—_R_elay-_a_ssisted _n_etwork-_c_oding (RANC) _a_utomatic _r_epeat re_q_uest (ARQ) protocols are ARQ protocols that leverage both opportunistic retransmission and network coding for wireless relay networks. This paper studies the issue of efficient network coding at relay for RANC ARQ. We develop a XOR-based with the help of Fibonacci sequence scheme, abbreviated as XOR Fibo. Simulation results show that in terms of relative inefficiency, XOR Fibo has a significant performance gain over a plain XOR scheme. Moreover, the use of XOR Fibo can provide close to the same performance as a random network coding scheme that uses a large field size, with the added advantage of requiring fewer and simpler operations during the encoding and decoding processes.

I. INTRODUCTION

To achieve reliable data transmission over inherently unreliable wireless channels, both relaying and *a*utomatic *r*epeat re*q*uest (ARQ) are techniques widely used in wireless networks. By relaying, an unreliable (or low-rate) direct communication between source and destination is replaced by two-hop transmission. With ARQ, the sender will resend a data packet if not receiving an acknowledgement (ACK), which indicates successful reception of a data packet. Recently, advanced techniques such as link-layer network coding [1],[2] and cooperative communications [3],[4] have been hot research topics to further improve network performance.

Combining the concept of the aforementioned techniques, Kao and Chen [5],[6] proposed the framework of relay-assisted network-coding (RANC) ARQ for one-way wireless relay networks. Fig. 1 gives a high-level overview of a RANC ARQ protocol. For each source-relay-destination triple, messages are transmitted on a *per-segment basis*. A *segment* consists of K *original blocks*, denoted by b_1, b_2, …, b_K, where the segment size K can be a constant or a positive integer valued random variable. The original blocks are not transmitted over wireless channels. Instead, a source node sends out *coded blocks*, denoted by b_1', b_2', …, which are produced, for example, by taking linear combinations of the original blocks over a finite field.

A relay node does not forward the coded blocks it overhears. Instead, when needed, the relay node produces a number of *recoded blocks*, denoted by b_1'', b_2'', …, for example, by taking linear combinations of all or a subset of the overheard coded blocks. After that, the recoded blocks are sent out to the destination node.

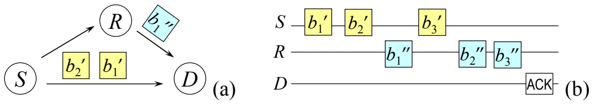

Fig. 1. (a) A three-node network containing source S, destination D, and relay R. The source node and relay node send out coded blocks, denoted by b_i', and recoded blocks, denoted by b_i'', respectively. (b) A high-level overview of RANC ARQ protocols.

Any transmission of coded/recoded block over wireless channels might fail to reach the destination. Regardless which blocks are lost, the destination node can retrieve the entire segment from the (coded and recoded) blocks it has received, as long as the destination has collected a sufficient number of blocks. Once the destination node retrieves the entire segment, a *segment transmission* completes. For RANC ARQ protocols, there is no need for the destination to send ACKs upon receptions of blocks; instead, a per-segment ACK is sent out by destination, notifying the source node of the termination of segment transmission.

A RANC ARQ protocol can be regarded as consisting of two functions—the cooperative communication (*CC*) function and the network coding (*NC*) function. The CC function involves cooperation among source, relay, and destination; in particular, it decides when to send out coded blocks from the source and recoded blocks from the relay. The NC function involves the algorithm of producing coded blocks at the source and recoded blocks at the relay.

There exists few research works investigating the CC function in RANC ARQ; examples include the two TDMA-based protocols proposed in [5] and the IEEE 802.11 compatible protocol introduced in [6]. However, the NC function in RANC ARQ has not been extensively explored yet. In particular, efficient network coding at relay still remains an open research topic, which is the focus of this paper.

This paper studies linear network coding at relay, with an emphasis on the schemes that only use exclusive-or (XOR) operations. A XOR-based network coding at relay scheme has the advantage of requiring fewer and simpler operations during the encoding process at relay and the decoding process at destination, compared with a random network coding (*RNC*) scheme that uses a large field size. However, a plain XOR-based network coding, shown by our simulation results, has a disadvantage—it has a performance away worse than RNC.

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To achieve comparable performance, we propose the XOR-based network coding with the help of Fibonacci sequence scheme, abbreviated as *XOR Fibo*, and show XOR Fibo has a performance close to RNC.

The remainder of this paper is organized as follows. Section II introduces the system model we consider in this paper. Sections III and IV present network coding at relay schemes existing in the literature and our proposed scheme, respectively. Section V evaluates and compares performance of these schemes through simulation. In Section 6, we present some concluding remarks.

II. SYSTEM MODEL

This section introduces the system model used in this paper to study the NC function in RANC ARQ. The NC function includes the ways to generate coded blocks at source and recoded blocks at relay. As explained later, while fixing the network coding at source scheme to always be the Vandermonde coding (VC) algorithm, we investigate several network coding at relay schemes, including a few schemes existing in the literature and a scheme we develop.

Similar to several papers such as [7], we consider only linear network coding and assume that a source node uses the VC algorithm to produce coded blocks. The VC algorithm is also known as the Reed-Solomon based coding algorithm.

The VC algorithm creates coded blocks over a finite field as:

$$b_i' = \sum_{j=1}^{s} r_i^{j-1} b_j \tag{1}$$

Each coded block b_i' has a different base r_i. The ordered set of coefficients, which is $(r_i^0, r_i^1, \ldots, r_i^{s-1})$ in the VC algorithm, is called the *coding vector* of the coded block b_i'.

The major reason of adopting the VC algorithm at source is its optimality, when there is no relaying mechanism. The VC algorithm guarantees that the coding vectors of any K coded blocks are linearly independent, which implies that all coded blocks received by the destination are innovative. Therefore, the VC algorithm is optimal in the sense that the destination can retrieve the entire segment once receiving K coded blocks, regardless of which coded blocks are received.

However, with relay nodes added in, the VC algorithm no longer guarantees the optimality; indeed, optimality for the NC function in RANC ARQ is undefined. The trivial definition—letting the optimal NC function be the network coding algorithm that minimizes the completion time (which is defined as the total number of transmissions sent by source and relay) until the destination node can retrieve the entire segment—is not self-contained. The reason for that is because link condition and the CC function in RANC ARQ affect the completion time significantly.

Link receptions are modeled by an erasure model in order to take into consideration the error-prone nature of wireless links. In this model, the reception probabilities of data packets (*i.e.*, *blocks*) are fractional numbers. Fig. 2 illustrates the link reception probabilities from source to destination (denoted by P_{SD}), from source to relay (denoted by P_{SR}), and from relay to destination (denoted by P_{RD}), respectively. It is assumed that the relay node is appropriately chosen and hence $P_{RD} \geq P_{SD}$.

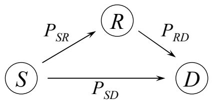

Fig. 2. Link reception probabilities of blocks (denoted by P_{SD}, P_{SR}, and P_{RD}), for a given source-destination pair.

The same CC function of RANC ARQ—the hold-and-proceed (HP) protocol [5]—is used to fairly evaluate several network coding at relay schemes presented in sections III and IV. The parameter h of the HP protocol is set to one. In this setup, a relay node generates and sends out a recoded block each time it overhears a coded block, except the first one. (For the first overheard coded block, the relay node simply stores the block in its buffer.)

How much different network coding at relay schemes affect the overall performance will be studied and the results will be shown later in Section V in a relative manner.

III. EXISTING NETWORK CODING AT RELAY SCHEMES

Whereas Section IV presents the network coding at relay scheme we propose, this section introduces several existing schemes used for performance comparison. As introduced in Section II, a source node always uses the VC algorithm to generate coded blocks b_1', b_2', …. The relay node overhears a subset of the coded blocks. Denote these overheard coded blocks by c_1', c_2', …. Given these overheard coded blocks, different network coding at relay schemes produce different sets of recoded blocks.

A. Forwarding

The Forwarding scheme simply forwards the overheard coded block without doing any further network coding at relay. In other words, the i-th recoded block the relay sends is

$$b_i'' = c_{i+1}' \tag{2}$$

because the first overheard coded block is not forwarded.

B. Plain XOR

The plain XOR scheme generates recoded blocks by taking XOR operations over all the overheard coded blocks stored in the relay's buffer. Denote the buffer size (in blocks) by B. This plain XOR scheme produces the i-th recoded block as:

$$b_i'' = \sum_{j=1}^{\max(i+1,B)} c_{i+2-j}' \tag{3}$$

C. Random network coding (RNC)

The random network coding (RNC) scheme produces recoded blocks by taking random network coding over all the overheard coded blocks stored in the relay's buffer. That is, the i-th recoded block is

$$b_i'' = \sum_{j=1}^{\max(i+1,B)} r_{i,j} c_{i+2-j}' \tag{4}$$

where $r_{i,j}$ is a number randomly chosen from a finite field and B is the the buffer size (in blocks).

IV. PROPOSED NETWORK CODING AT RELAY SCHEME

To achieve a performance comparable to RNC using a XOR-based network coding at relay, we propose the XOR-based network coding with the help of Fibonacci sequence scheme, abbreviated as *XOR Fibo*. Different from the plain XOR scheme introduced in III.B, the relay node does not take exclusive-or operations over all the overheard coded blocks stored in the relay's buffer:

$$b_i'' \neq c_{i+1}' \oplus c_i' \oplus c_{i-1}' \oplus c_{i-2}' \oplus \cdots$$

Instead, there are gaps in the overheard blocks over which the relay does not take exclusive-or operations:

$$b_i'' = c_{i+1}' \oplus c_i' \oplus c_{(i-1)-F(1)}' \oplus c_{(i-2)-F(1)-F(2)}' \oplus c_{(i-3)-F(1)-F(2)-F(3)}' \oplus \cdots \quad (5)$$

When the relay's buffer is not full, the lengths of these gaps follow the Fibonacci sequence modulo the buffer size B. The Fibonacci sequence is 1, 1, 2, 3, 5, 8, …. The length of the n-th gap is set to $F_n = (F_{n-1} + F_{n-2}) \bmod B$, where the first two numbers in the sequence are $F_1 = 1$ and $F_2 = 1$. We name the sequence $\{F_n,\ n = 0, 1, 2, \ldots\}$ the modular Fibonacci sequence.

Let us give an example. Suppose that the size of the relay's buffer is $B = 10$ and the relay has overheard six coded blocks, $c_1', c_2', \ldots, c_6'$. Then because the first few numbers in the modular Fibonacci sequence are $F_1 = 1$ and $F_2 = 1$, the latest recoded block the relay produces is

$$b_5'' = c_6' \oplus c_5' \oplus c_3' \oplus c_1'$$

where this example has two gaps over which the relay does not take exclusive-or operations.

When the relay's buffer is full, the modular Fibonacci sequence aforementioned is modified slightly: We change the first number in the modular Fibonacci sequence to be $F_1 = 1 + \lceil M_R / B \rceil \bmod B$ and the second number to be $F_2 = 1 + M_R \bmod B$, where M_R is the number of coded blocks the relay node has overheard. The remaining numbers in the modular Fibonacci sequence can be obtained by the same recursive expression, $F_n = (F_{n-1} + F_{n-2}) \bmod B$.

V. PERFORMANCE EVALUATION

In this section, we compare by simulation the performances of the four network coding at relay schemes described in sections III and IV—Forwarding, plain XOR, XOR Fibo, and RNC. The Forwarding scheme does not apply network coding at relay and thus is expected to perform worst from the communication perspective. Plain XOR is a trivial XOR-based scheme and XOR Fibo is an advanced XOR-based scheme we propose to perform comparably to non-XOR-based linear network coding schemes. The RNC scheme is not based on XOR operations because it uses a large field size. A non-XOR-based scheme using a large field size is expected to perform best, at a cost of requiring more and harder operations during the encoding and decoding processes.

To quantify the performance gain of our proposed scheme compared with a trivial XOR-based scheme and to learn its performance loss compared with a non-XOR-based scheme, we run simulation and present two performance metrics for each of the schemes aforementioned. The first performance metric is the number of non-innovative blocks received by the destination node within a segment transmission. This performance metric is abbreviated as *the number of non-innovative blocks*. An efficient network coding at relay scheme should cause as few non-innovative blocks as possible.

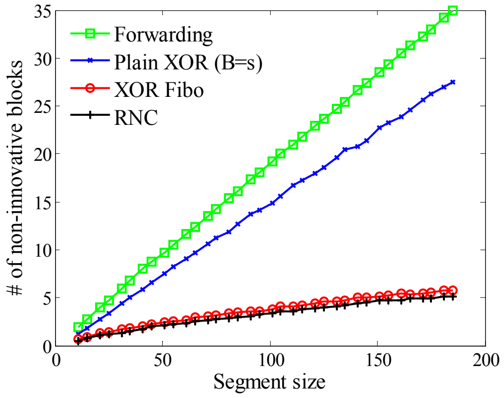

Fig. 3. The average number of non-innovative blocks (received by the destination within a segment transmission) vs. the segment size when P_{SD} = 0.2, P_{SR} = 1, and P_{RD} = 0.8.

The second performance metric for a given network coding at relay scheme is the inefficiency relative to RNC (or abbreviated as *relative inefficiency*). It is defined as:

$$\text{Inefficiency relative to RNC} = \frac{M_D(\cdot)}{M_D(\text{RNC})} - 1 \quad (6)$$

where $M_D(\cdot)$ is the total number of blocks received by the destination node within a segment transmission for the given scheme. The relative inefficiency is the smaller the better.

We evaluate the performances of the aforementioned schemes under two sets of environments. In the first simulation setup, we set P_{SD} = 0.2, P_{SR} = 1, and P_{RD} = 0.8. The simulation results of the number of non-innovative blocks and the relative inefficiency are shown in Fig. 3 and Fig. 4, respectively.

As one can see in Fig. 3 and Fig. 4, the Forwarding scheme performs worst. The plain XOR scheme performs better than the Forwarding scheme and XOR Fibo performs significantly better than the plain XOR scheme. Indeed, XOR Fibo

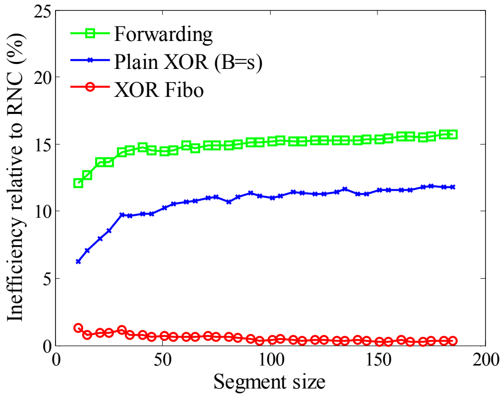

Fig. 4. The inefficiency relative to RNC vs. the segment size when P_{SD} = 0.2, P_{SR} = 1, and P_{RD} = 0.8.

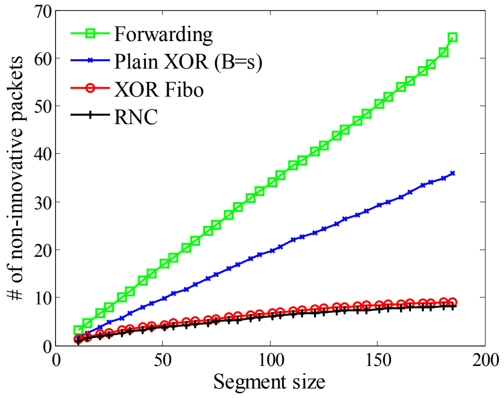

Fig. 5. The average number of non-innovative blocks (received by the destination within a segment transmission) vs. the segment size when P_{SD} = 0.5, P_{SR} = 1, and P_{RD} = 0.5.

performs almost equally well, compared to RNC. This can be easily observed in Fig. 3 and Fig. 4.

In addition, as shown in Fig. 4, the relative inefficiency for the Forwarding scheme is roughly 16%. This implies that roughly 16% of the blocks the destination receives are non-innovative, assuming that RNC is an ideal scheme. This quantity can be explained as follows. The Forwarding scheme does not apply network coding at relay and thus the relay sends out a replica for each block it overhears. The replica, supposing it is successfully delivered to the destination, is non-innovative if and only if the corresponding direct communication from source to destination succeeds. So among all the blocks the destination receives, including those transmitted over the direct link and those transmitted via relay, the proportion of non-innovative blocks under the Forwarding scheme is roughly

$$\frac{P_{SD}\cdot P_{SR}P_{RD}}{P_{SD}+P_{SR}P_{RD}} \qquad (7)$$

The denominator of (7) consists of two terms; these two terms correspond to the blocks delivered over the direct link and via relay, respectively. Substituting P_{SD} = 0.2, P_{SR} = 1, and P_{RD} = 0.8 into (7), we obtain its value to be 0.16, which is consistent with the simulation results in Fig. 4 for the Forwarding scheme.

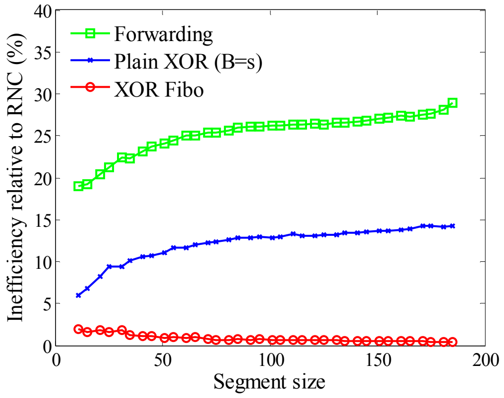

Fig. 6. The inefficiency relative to RNC vs. the segment size when P_{SD} = 0.5, P_{SR} = 1, and P_{RD} = 0.5.

In the second simulation setup, we set P_{SD} = 0.5, P_{SR} = 1, and P_{RD} = 0.5. The simulation results of the number of non-innovative blocks and the relative inefficiency in this setup are shown in Fig. 5 and Fig. 6, respectively. We note that all the qualitative and quantitative observations mentioned above for the first simulation setup also apply to this simulation setup. In particular, the Forwarding still performs worst and its relative inefficiency is roughly equal to the value computed by (7). The plain XOR scheme still performs better than the Forwarding scheme. The XOR Fibo scheme still outperforms the plain XOR scheme and still performs almost equally to the RNC scheme.

VI. CONCLUSIONS

In this paper, we have addressed the two functions which a RANC ARQ protocol consists of, namely, the cooperative communication (*CC*) function and the network coding (*NC*) function. This paper aims to study the network coding function, with an emphasis on linear network coding at relay. To take fewer and simpler operations during the encoding and decoding processes, we have developed the XOR Fibo scheme, which takes XOR operations only rather than using a large field size. Our simulation results show that although XOR Fibo is a XOR-based scheme, it outperforms the plain XOR scheme drastically and performs comparably well to the random network coding scheme that uses a large field size.

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