CS5321 Numerical Optimization Homework 2

One dimensional problems

Due March 19, 2012

1. (40%) A function f is called *convex* if

$$f(\alpha x_1 + (1-\alpha)x_2) \leq \alpha f(x_1) + (1-\alpha) f(x_2)$$

for any x_1, x_2 in the interval of interests, and $\alpha \in [0,1]$.

(a) Prove that for a convex function, any local minimizer is a global minimizer.

(b) Prove that any stationary point of a convex function is a minimizer.

(c) Prove that a twice differentiable function is convex on an interval if and only if its second derivative is non-negative everywhere.

Hint: For the if part: use Taylor theorem (with reminder) and the mean value theorem of derivative, which is for a differentiable function f and $x > z > y$,

$$f'(z) = \frac{f(x) - f(y)}{x - y}.$$

For the only if part: use this definition of second derivative

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

2. For $x \in [0,1]$, $f(x) = x^2 \sin\left(\frac{8}{x}\right), f(0) = 0$.

(a) (20%) Compute f' and f''. (Submit this part on 3/8 in class to make sure everyone has the right formula to continue.)

(b) (40%) Implement a Matlab code to find one of its local minimizer by line search method using $x_0 = 0.6, 0.3$, and 0.1. Describe the algorithm you used to compute the search directions and step lengths.

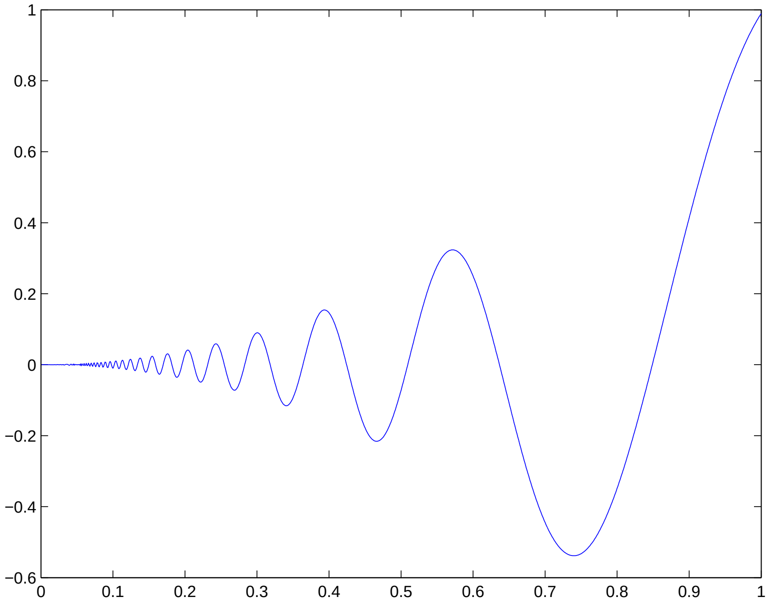

Figure 1: $f(x) = x^2 \sin\left(\frac{8}{x}\right), f(0) = 0$