

# CS5321 Numerical Optimization Homework 4

Due April 25

1. (10%) What is the distance of a point  $\vec{p}$  to a hyperplane  $\vec{a}^T \vec{x} + b = 0$ . Justify your answer.
2. (40%) Our frequently used matrix norms are called *subordinate matrix norm* because they are derived from corresponding vector norms. For an  $n \times m$  matrix  $A$ , its 1-norm, 2-norm and infinite-norm are defined by

$$\|A\|_p = \max_{\|x\|_p=1} \|Ax\|_p,$$

where  $p = 1, 2, \infty$  respectively.

- (a) What is the matrix 1-norm? Justify your answer?
  - (b) What is the matrix  $\infty$ -norm? Justify your answer?
  - (c) What is the matrix 2-norm? Justify your answer?
  - (d) Show the condition number of an invertible matrix  $A$ ,  $\kappa(A)$ , equations to  $\sigma_1/\sigma_n$ , where  $\sigma_1$  is the largest singular value of  $A$  and  $\sigma_n$  is the smallest singular value of  $A$ .
3. (50%) Consider the following linear program:

$$\begin{aligned} \max_{x_1, x_2} \quad & z = 8x_1 + 5x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 1000 \\ & 3x_1 + 4x_2 \leq 2400 \\ & x_1 + x_2 \leq 700 \\ & x_1 - x_2 \leq 350 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- (a) Transform it the standard form.
- (b) Suppose the initial guess is  $(0, 0)$ . Use the simplex method to solve this problem. In each iterations, show
  - Basic variables and non-basic variables, and their values.
  - Pricing vector.
  - Search direction.
  - Ratio test result.