CS5321 Numerical Optimization Homework 1

Due March 11

1. (70%) Figure 1 shows the plot of a function $f(x)$. Figure 2 shows the Matlab code and the convergence result of the bisection method.

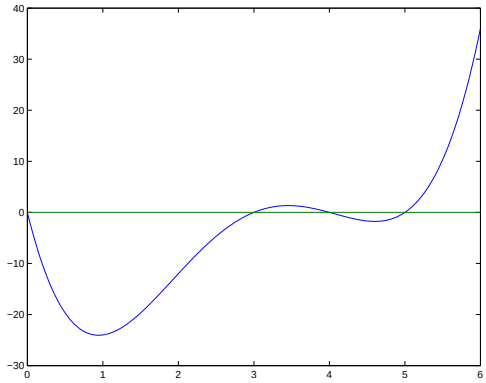

Figure 1: Function plot of $f(x) = x^4 - 12x^3 + 47x^2 - 60x$.

   (a) (10%) The convergent result of the bisection method, as shown in figure 2, has zigzag patterns. Can you explain this phenomenon?

   (b) (50%) Write Matlab codes to implement Newton's method (using initial guess $x_0 = 0$) and the interpolation method (using initial points $x_1 = 0, x_2 = 6, x_3 = 2.5$) to solve the same problem. Compare their convergent results to the that of the bisection method.

   (c) (10%) What happens if the interpolation method starts with $x_1 = 0, x_2 = 6, x_3 = 3$? How to avoid such situation?

2. (30%) It is known that Newton's method converges linearly when the equation has repeated roots.

   (a) (15%) Write Matlab codes to compare the the convergence of solving $f(x) = (x^2 - 1)(x + 6) = 0$ and solving $g(x) = (x - 1)^2(x + 6) = 0$ using Newton's method with initial guess $x_0 = 2$.

   (b) (15%) The rescue of this problem is to make the $x_{k+1} = x_k - 2\frac{f(x_k)}{f'(x_k)}$.[1] Use this formula to solve $g(x) = (x-1)^2(x+6) = 0$ with initial guess $x_0 = 2$. Compare the convergent result to the one in 2(a).

[1] The derivation can be found in `http://www.cs.nthu.edu.tw/~cherung/teaching/2008cs3331/lec02.pdf` and Stewart's afternote.

```
function bisection
% This program uses the bisection method to find the minimum of
%     f(x) = x^4-12*x^3+47*x^2-60*x.
% The feasible interval is [0,6].
% ================================================================

% Here we use the Matlab's bulidin function 'fzero' to find the
% "real solution", which will be used to measure errors.
sol = fzero(@gx,1);

% Initial
a = 0;
b = 6;
tol = 1e-5;  % the approximation within the tolerance.
hist = [];
while (b-a) >tol
    c = (a+b)/2;
    g_c = gx(c);

    hist = [hist;abs(c-sol)];     % bookkeeping the error

    % decide the direction
    if (g_c > 0), b = c;
    else          a = c;
    end
end

% plot the convergence
n = size(hist);
semilogy(1:n,hist, 'o-');
xlabel('Iterations');
ylabel('Error');

% the nested function is used to calculate g(x)=f'(x)
    function y=gx(x)
        y = 4*x^3-36*x^2+94*x-60;
    end
end
```

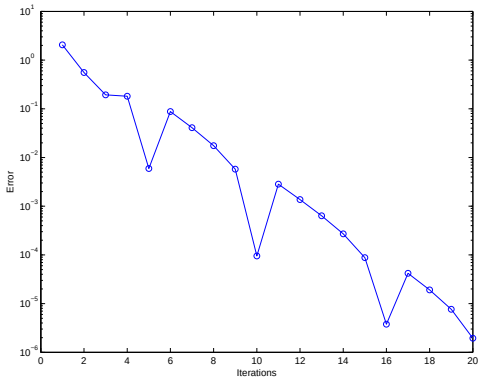

Figure 2: Matlab code and the convergent result of the bisection method.