CS3331 Numerical Methods

Quiz 8, Dec 30

Name: _____, ID: _____

1. Given a vector $\mathbf{y} = [1\ 3\ 0\ 7\ 4\ 2\ 6\ 5]^T$.

   (a) Rearrange the elements of $\mathbf{y}$ in the bit-reverse order. (10pt)

   The bit-reverse order is $[0, 4, 2, 6, 1, 5, 3, 7]$.
   So the reversed vector becomes $[1\ 4\ 0\ 6\ 3\ 2\ 7\ 5]^T$

   (b) The non-recursive FFT processes the bit-reversed vector in $O(\log_2(n))$ loops. What does the vector become after one loop? (10pt)

$$\begin{pmatrix} 1+4 \\ 1-4 \\ 0+6 \\ 0-6 \\ 3+2 \\ 3-2 \\ 7+5 \\ 7-5 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 6 \\ -6 \\ 5 \\ 1 \\ 12 \\ 2 \end{pmatrix}$$

2. $f(x) = c_1 \cos(x) + c_2 \left| \frac{2x}{\pi} - 1 \right|$ approximates $(0,1), (\frac{\pi}{2}, 2), (\pi, 3), (\frac{3\pi}{2}, 4)$ in the least square sense.

   (a) Write down the formula of normal equation and the matrix $\mathbf{A}$ of this problem. (10pt)

   Normal equation: $\mathbf{A}^T\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{b}$

$$\mathbf{A} = \begin{pmatrix} \cos(0) & |\frac{2}{\pi}0 - 1| \\ \cos(\pi/2) & |\frac{2}{\pi}\frac{\pi}{2} - 1| \\ \cos(\pi) & |\frac{2}{\pi}\pi - 1| \\ \cos(3\pi/2) & |\frac{2}{\pi}\frac{3\pi}{2} - 1| \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}$$

   (b) What are c_1 and c_2? (10pt)

$$\mathbf{A}^T\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix}, \mathbf{A}^T\mathbf{b} = \begin{pmatrix} -2 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

   (c) What is the least square error? (10pt)

$$\mathbf{b} - \mathbf{A} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

   Least square error is $0 + 2^2 + 0 + 0 = 4$