CS3331 Numerical Methods

Quiz 7, Dec 19

Name: _____, ID: _____

1. Compute the quadratic polynomial that interpolates the data points,

$$(x, f(x)) = (0, 2), (1, 5), (2, 3)$$

(The polynomial need not be expanded.)

(a) Use Lagragnge form. (10pt)

$$p(x) = 2\frac{(x-1)(x-2)}{(0-1)(0-2)} + 5\frac{(x-0)(x-2)}{(1-0)(1-2)} + 3\frac{(x-0)(x-1)}{(2-0)(2-1)}$$

$p(x) = -5/2x^2 + 11/2x + 2$ if you expand it.

(b) Use divided difference method. (10pt)

0	2		
1	5	$\frac{5-2}{1-0} = 3$	
2	3	$\frac{3-5}{2-1} = -2$	$\frac{-2-3}{2-0} = -5/2$

$$p(x) = 2 + 3(x-0) + (-5/2)(x-0)(x-1)$$

$p(x) = -5/2x^2 + 11/2x + 2$ if you expand it.

2. How many unknowns and how many equations are given if the 4th degree piecewise polynomials, $f_i(x) = a_{i,4}x^4 + a_{i,3}x^3 + a_{i,2}x^2 + a_{i,1}x + a_{i,0}$, are used to interpolate n data points, $n \geq 2$, such that the function value, first derivative, second derivative, and third derivatives are continuous? (10pt).

Unknowns: $5(n-1)$

- $n-1$ polynomials, and each has 5 unknowns.

Equations: $5n-8$

- Continuity of function values: $2(n-1)$
- Continuity of first, second, and third derivatives: $(n-2)+(n-2)+(n-2)$

3. Suppose a polynomial

$$p(x) = c_0 + c_1x + c_2x^2 + c_3x^3$$

interpolates data points $(x, f(x)) = (0,1), (1,5), (2,3)$, and the derivative at 1, $f'(1) = 2$. What are c_0, c_1, c_2, and c_3? (20pt)

There are many ways to compute it. Here uses the divided differences

0	1			
1	5	$\frac{5-1}{1-0} = 4$		
1	5	$f'(1) = 2$	$\frac{2-4}{1-0} = -2$	
2	3	$\frac{3-5}{2-1} = -2$	$\frac{-2-2}{2-1} = -4$	$\frac{-4-(-2)}{2-0} = -1$

$$\begin{aligned} p(x) &= 1 + 4(x-0) - 2(x-0)(x-1) - 1(x-0)(x-1)^2 \\ &= 1 + 5x + 0x^2 - x^3 \end{aligned}$$

$c_0 = 1, c_1 = 5, c_2 = 0, c_3 = -1$