Solutions for P3+P4 of Exam, 2020

Due by 15:50, December 16, 2020

1.(10%) Apply $LU-decomposition$ with partial pivoting and back substitution to solve the following linear system of equations such that the accuracy is within 10^{-4}.

$$\begin{array}{ccccccc} x & + & \frac{1}{2}y & + & \frac{1}{3}z & = & 1 \\ \frac{1}{2}x & + & \frac{1}{3}y & + & \frac{1}{4}z & = & 1 \\ \frac{1}{3}x & + & \frac{1}{4}y & + & \frac{1}{5}z & = & 1 \end{array}$$

Ans: $[3.0000; \; -24.0000; \; 30.0000]$

2.(20%) Find the characteristic polynomial of the matrix A given below and compare the roots of the characteristic equation of A with those obtained from the matlab command $eig(A)$.

$$A = \begin{bmatrix} -7 & 3 & 1 \\ 3 & -7 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

Ans: $P(x) = x^3 + 9x^2 - 35x - 247$, $[-10.0350; \; -4.4707; \; 5.5056]$

3.(10%) The nonlinear system of equations with variables $x = X(1), y = X(2), z = X(3)$ to be solved is

$$\begin{array}{ccccccc} xy & - & z^2 & & & = & 1 \\ xyz & - & x^2 & + & y^2 & = & 2 \\ e^x & - & e^y & - & e^z & = & 3 \end{array}$$

Ans: $[2.1659; \; 1.4493; \; 1.4625]$

4. For $x \in (-1,1)$, define the Chebyshev polynomial of degree n by

$$T_n(x) = cos(ncos^{-1}x) \ \forall \ n \geq 0$$

Denote the inner product by $\langle T_n, T_m \rangle = \int_{-1}^{1} T_n(x)T_m(x)\frac{1}{\sqrt{1-x^2}}dx$

(a) $T_0(x) = 1,\ T_1(x) = x,\ T_2(x) = 2x^2 - 1,\ T_3(x) = 4x^3 - 3x,\ T_4(x) = 8x^4 - 8x^2 + 1.$

(b) Refer to class lectures.

(c) $\langle T_0, T_0 \rangle = \pi$ and $\langle T_n, T_n \rangle = \frac{\pi}{2}$ for $n \geq 1$.

(d) $cos(\frac{\pi}{n}(k + \frac{1}{2})),\ n \in N,\ k \in Z$

5.

(b) $0.5 \cdot \frac{(x-2.5)(x-4)}{(2-2.5)(2-4)} + 0.4 \cdot \frac{(x-2)(x-4)}{(2.5-2)(2.5-4)} + 0.25 \cdot \frac{(x-2)(x-2.5)}{(4-2)(4-2.5)}$

(c) $0.5 - 0.2 \cdot (x-2) + 0.05 \cdot (x-2)(x-2.5)$

(d) $0.05 \cdot x^2 - 0.425 \cdot x + 1.15$

6.(20%) A natural cubic spline S on [0,2] is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & if\ 0 \leq x < 1 \\ S_1(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3, & if\ 1 \leq x \leq 2 \end{cases}$$

Find $a, b, c,\ and\ d$, respectively.

(Answer) $(a, b, c, d) = (2, -1, -3, 1)$