Solutions for HW3

(1) Verify the following Sherman-Morrison formula:

$$(A+uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}$$

is valid provided that $1+v^TA^{-1}u \neq 0$.

Proof: Suppose that $1+v^TA^{-1}u \neq 0$ and $A+uv^T$ is invertible, we shall prove that

$$(A+uv^T)(A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}) = I, \ \textit{where } I \textit{ isan } n \times n \textit{ identity matrix.}$$

Note that

$$\begin{aligned}
& (A+uv^T)(A^{-1} - \tfrac{A^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}) \\
= \ & I + uv^TA^{-1} - \tfrac{uv^TA^{-1}}{1+v^TA^{-1}u} - \tfrac{uv^TA^{-1}uv^TA^{-1}}{1+v^TA^{-1}u} \\
= \ & I + uv^TA^{-1} - \tfrac{uv^TA^{-1}}{1+v^TA^{-1}u} - \tfrac{u(\mathbf{v^TA^{-1}u})v^TA^{-1}}{1+v^TA^{-1}u} \\
= \ & I + uv^TA^{-1} - \tfrac{uv^TA^{-1}}{1+v^TA^{-1}u} - \tfrac{(\mathbf{v^TA^{-1}u})uv^TA^{-1}}{1+v^TA^{-1}u}, \quad \textit{since } (\mathbf{v^TA^{-1}u}) \textit{ is a scalar} \\
= \ & I + uv^TA^{-1} - \tfrac{1+(\mathbf{v^TA^{-1}u})}{1+v^TA^{-1}u}uv^TA^{-1} \\
= \ & I + uv^TA^{-1} - uv^TA^{-1} \\
= \ & I
\end{aligned}$$

(2) I need to read HW2 again today or tomorrow evening, but it shoud not be that hard.
Could you provide me your solution for Problem 6 of HW2?