H2: Maximum Likelihood Estimators

1. Let $X_1, X_2, \cdots, X_n$ be a random sample size $n$ of Poisson distribution with mean $\lambda$, that is,
   \[ f_i(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \ x = 0, 1, \cdots \]
   Find the maximum likelihood estimator of $\lambda$.

2. Let $X_1, X_2, \cdots, X_n$ be a random sample size $n$ of Normal distribution with mean $\mu$ and variance $\sigma^2$, that is,
   \[ f_i(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}, \ -\infty < x < \infty \]
   Find the maximum likelihood estimator of $\mu$ and $\sigma^2$.

3. Let $X_1, X_2, \cdots, X_n$ be a random sample size $n$ of Exponential distribution with mean $\theta$, that is,
   \[ f_i(x) = \frac{1}{\theta} e^{-x/\theta}, \ x \geq 0 \]
   Find the maximum likelihood estimator of $\theta$.

4. Let $X_1, X_2, \cdots, X_n$ be a random sample size $n$ of Geometric distribution with parameter $p$, that is,
   \[ f_i(x) = (1-p)^{x-1}p, \ x = 1, 2, \cdots \]
   Find the maximum likelihood estimator of $p$.

5. Let $X_1, X_2, \cdots, X_n$ be a random sample size $n$ of $\chi^2$ distribution with $r$ degrees of freedom, that is,
   \[ f_i(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{(r/2)-1} e^{-x/2}, \ x > 0 \]
   Find the maximum likelihood estimator of $r$. 