H5: Eigenvalues/Eigenvectors

1. Let $A \in R^{n\times n}$ have eigenvalues $2, 4, \cdots, 2n$. Show that $tr(A) = n(n+1)$ and $det(A) = 2^n \cdot n!$.

2. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$,

(a) Show that the characteristic polynomial of A is $x(x-1)^2$.

(b) Find the eigenvalues, eigenvectors of matrix A, and the corresponding eigenspaces.

3. For the matrix B given in problem **2**,

(a) Find the eigenvalues and eigenvectors of B.

(b) Calculate $\|B\|_1$, $\|B\|_\infty$, $\|B\|_2$.

(c) Give a spectrum decomposition of matrix B.

4. Verify your solutions for problems **2.** and **3.** by using Matlab.

Solutions for H5: Eigenvalues/Eigenvectors

1. $tr(A) = \sum_{k=1}^{n}(2k) = n(n+1)$, $det(A) = \Pi_{k=1}^{n}(2k) = 2^n \cdot n!$

2. `(a) poly(A),  (b) [U, D]=eig(A)`

3.

(a) $\lambda_1 = -1$, $\mathbf{u}_1 = \frac{1}{\sqrt{2}}[1,1,0]^t$, $\lambda_2 = -3$, $\mathbf{u}_2 = \frac{1}{\sqrt{2}}[1,-1,0]^t$, $\lambda_3 = 2$, $\mathbf{u}_3 = [0,0,1]^t$,

(b) $\|B\|_1 = 3$, $\|B\|_2 = 3$, $\|B\|_\infty = 3$

(c) $B = UDU^t$, where $U = [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$, $D = diag(-1,-3,2)$

(d) $\sigma_1 = 3$, $\sigma_2 = 2$, $\sigma_3 = 1$, $B = USV^t$, where $S = diag(3,2,1)$, and $U = [\mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_1]$, $V = [-\mathbf{u}_2, \mathbf{u}_3, -\mathbf{u}_1]$,

(e) $e^B = Udiag(e^{-1}, e^{-3}, e^2)U^t$

4(a∼b) `[U, D]=eig(B);  norm(B,1), norm(B,2), norm(B, inf)`

4(c∼d) `norm(B-U*D*U',2) = 0? ; [U S V]=svd(B)`