Solutions for Test II

1. Write down the following probability density functions and compute their moment generating functions.

(a) $f(x) = C(50, x)(0.6)^x(0.4)^{50-x},\ 0 \le x \le 50;\quad M(t) = (0.4 + 0.6e^t)^{50}$

(b) $f(x) = e^{-4}4^x/x!,\ x = 0, 1, 2, \cdots;\quad M(t) = e^{4(e^t-1)}$

(c) $f(x) = \frac{1}{2}e^{-x/2},\ x > 0;\quad M(t) = \frac{1}{1-2t}$

(d) $f(x) = \frac{1}{2\cdot\sqrt{2\pi}}e^{-(x-3)^2/8},\quad -\infty < x < \infty;\quad M(t) = e^{3t+2t^2}$

(e) $f(x) = \frac{1}{\Gamma(12/2)2^6}x^5 e^{-x/2},\ x \ge 0;\quad M(t) = \frac{1}{(1-2t)^6}$

2(a) $E(X) = \frac{1}{2},\ Var(X) = \frac{1}{4} - \frac{1}{\pi^2}.$

2(b) $F(X) = \frac{1}{2}(1 - \cos \pi x),\ 0 \le x \le 1.$

2(c~e) $x_{0.25} = \frac{1}{3},\ median\ = \frac{1}{2},\ q_3 = \frac{2}{3}$

3(a) $P(A_i) = \frac{4!}{5!},\ \forall\ 1 \le i \le 5.$

3(b) $P(A_i \cap A_j) = \frac{3!}{5!}$, where $1 \le i < j \le 5$.

3(c) $P(A_i \cap A_j \cap A_k) = \frac{2!}{5!}$, where $1 \le i < j < k \le 5$.

3(d) $P(A_i \cap A_j \cap A_k \cap A_m) = \frac{1!}{5!}$, where $1 \le i < j < k < m \le 5$.

3(e) $P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) = 1 - \frac{1!}{5!} + \frac{2!}{5!} - \frac{3!}{5!} + \frac{4!}{5!} - \frac{5!}{5!}$

4. $P(Y \le y) = P(F(X) \le y) = P(X \le F^{-1}(y)) = F(F^{-1}(y)) = y$

5(a) $Y \sim b(2000, \frac{1}{4}, \frac{3}{4}).$

5(b) $E(Y) = 500,\quad Var(X) = 375.$

5(c) $E(Y/500) = 1.$

5(d) $P(Y \le 100) = \sum_{k=0}^{100} \binom{2000}{k} (\frac{1}{4})^k(\frac{3}{4})^{2000-k}.$

6. $E(X) = 24,\ Var(X) = 48$, where $X \sim \chi^2(24)$.

7. (a) $f(y) = \frac{1}{2}e^{-y/2},\ y > 0;$ (b) $Exponential(2)$

8. (a) $(1 - p + pe^t)^{n_1+n_2},$ (b) $b(n_1 + n_2, p)$

9. $\prod_{i=1}^{n} e^{\lambda_i(e^t-1)} = e^{\lambda(e^t-1)}$, where $\lambda = \sum_{i=1}^{n} \lambda_i$

10. $W \sim \chi^2(n)$