

## Homework 4: Continuous Distributions

(1) Let a r.v.  $X$  have the probability density function  $f(x) = \frac{1}{2}\sin(x)$ ,  $0 \leq x \leq \pi$ .

- (a) Find the mean  $\mu$  and variance  $\sigma^2$ .
- (b) Sketch the graph of the p.d.f. of  $X$ .
- (c) Sketch the graph of the distribution function of  $X$ .

(2) Let  $X, Y$  be a random sample of size 2 from  $\sim N(3, 0.25)$ . Define  $Z = 2(X - 3)$ ,  $U = 2(Y - 3)$ ,  $W = Z^2$ ,  $V = Z + U$ .

- (a) Write down the probability density function of  $X$ .
- (b) Show that  $Z$  has the standard normal distribution.
- (c) What is the moment-generating function of  $Z$ ?
- (d) Show that  $W \sim \chi^2(1)$ .
- (e) What is the moment-generating function of  $W$ ?
- (f) What is the moment-generating function of  $V$ ?
- (g) How is  $V$  distributed?
- (h) What is the probability density function of  $V$ ?

(3) Let  $X$  have an exponential distribution with a mean of  $\theta = 20$ . Compute

- (a)  $P(10 < X < 30)$
- (b)  $P(X > 30)$
- (c)  $P(X > 40 | X > 10)$

(4) Plot the following exponential density functions in a single frame.

- (a) An exponential function with mean 1.
- (b) An exponential function with mean 2.
- (c) An exponential function with mean 4.
- (d) An exponential function with mean 7.

**(5)** Plot the following  $\chi^2(r)$  density functions in a single frame.

- (a)  $\chi^2(1)$ .
- (b)  $\chi^2(2)$ .
- (c)  $\chi^2(4)$ .
- (d)  $\chi^2(7)$ .

**(6)** Plot the following normal density functions in a single frame.

- (a)  $X \sim N(0, 1^2)$
- (b)  $X \sim N(0, 2^2)$
- (c)  $X \sim N(0, (2.5)^2)$
- (d)  $X \sim N(0, 3^2)$