

Homework 4: Continuous Distributions

- (1) Let a r.v. X have the probability density function $f(x) = \frac{1}{2}\sin(x)$, $0 \leq x \leq \pi$.
- (a) Find the mean μ and variance σ^2 .
 - (b) Sketch the graph of the p.d.f. of X .
 - (c) Sketch the graph of the distribution function of X .
- (2) Let X, Y be a random sample of size 2 from $\sim N(3, 0.25)$. Define $Z = 2(X - 3)$, $U = 2(Y - 3)$, $W = Z^2$, $V = Z + U$.
- (a) Write down the probability density function of X .
 - (b) Show that Z has the standard normal distribution.
 - (c) What is the moment-generating function of Z ?
 - (d) Show that $W \sim \chi^2(1)$.
 - (e) What is the moment-generating function of W ?
 - (f) What is the moment-generating function of V ?
 - (g) How is V distributed?
 - (h) What is the probability density function of V ?
- (3) Let X have an exponential distribution with a mean of $\theta = 20$. Compute
- (a) $P(10 < X < 30)$
 - (b) $P(X > 30)$
 - (c) $P(X > 40 | X > 10)$
- (4) Plot the following exponential density functions in a single frame.
- (a) An exponential function with mean 1.
 - (b) An exponential function with mean 2.
 - (c) An exponential function with mean 4.
 - (d) An exponential function with mean 7.

(5) Plot the following $\chi^2(r)$ density functions in a single frame.

(a) $\chi^2(1)$.

(b) $\chi^2(2)$.

(c) $\chi^2(4)$.

(d) $\chi^2(7)$.

(6) Plot the following normal density functions in a single frame.

(a) $X \sim N(0, 1^2)$

(b) $X \sim N(0, 2^2)$

(c) $X \sim N(0, (2.5)^2)$

(d) $X \sim N(0, 3^2)$