1. (10%) Bean seeds from supplier B have an 85% germination rate and those from supplier C have a 75% germination rate. A seed package company purchases 40% of their bean seeds from supplier B and 60% from supplier C and mixes these seeds together.

(a) Find a probability that a seed selected at random from the mixed seeds will germinate, that is, compute \( P(G) \).
(b) Given that a seed germinates, find the probability that the seed was purchased from supplier B.
(c) Given that a seed germinates, find the probability that the seed was purchased from supplier C.

2. (10%) Let the random variable \( X \) have a p.d.f.

\[
f(x) = \frac{(|x| + 1)^2}{9}, \quad x = -1, 0, 1
\]

Compute \( E(X) \), \( E(X^2) \), and \( E(3X^2 - 2X + 4) \).

3. (10%) It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let \( X \) equal the number with no health insurance in a random sample of \( n = 15 \) Americans.

(a) How is \( X \) distributed?
(b) Give the mean, variance, and standard deviation of \( X \).
(c) Find \( P(X \geq 5) \).

4. (10%) Let \( W \) have a geometric distribution with parameter \( p \).

(a) Give the probability density function of \( W \).
(b) Show that \( P(W > (k + j)|W > k) = P(W > j) \), where \( k, j \) are nonnegative integers.
5. (10%) Write down the following probability density functions and give their moment-generating functions.

(a) Binomial distribution with mean 30 and variance 12.
(b) Poisson distribution with variance 4.
(c) Exponential distribution with variance 4.
(d) Normal distribution with mean 3, variance 4.
(e) $\chi^2$ distribution with the degrees of freedom 2.

6. (10%) Explain what the following matlab codes do in each problem.

(a) $X=0:10; \ Y=\text{binopdf}(X,10,0.8); \ \text{bar}(X,Y,0.8)$

(b) $X=1:10; \ Y=\text{geopdf}(X,0.5); \ \text{bar}(X,Y,0.8)$

(c) $X=0:0.1:10; \ Y=0.5*\exp(-X/2); \ \text{plot}(X,Y,'-')$

(d) $X=0.1:0.1:8; \ Y=\text{chi2pdf}(X,4); \ \text{plot}(X,Y,'-')$

(e) $X=-3:0.1:5; \ Y=\text{normpdf}(X,1,2); \ \text{plot}(X,Y,'-')$
7. (20%) Let $X$ be a discrete random variable (r.v.) having the probability mass function (p.m.f. or p.d.f.) $f(x)$, then the mean $\mu$, variance $\sigma^2$, and the corresponding moment-generating function $\phi(t)$ are defined as follows.

$$\mu = E[X] = \sum_x x f(x)$$

$$\sigma^2 = Var(X) = E[(X - \mu)^2] = E[X^2] - (E[X])^2$$

$$\phi(t) = E[e^{tx}] = \sum_x e^{tx} f(x)$$

Moreover, we know that

$$\mu = \phi'(0), \quad \text{and} \quad \sigma^2 = \phi''(0) - [\phi'(0)]^2$$

For a discrete type of r.v. $X$ which has one of the following probability mass functions, *derive* the formula for the moment-generating function, and compute the mean and variance, respectively.

**Binomial** $f(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$, $x = 0, 1, 2, \ldots, n$

**Geometric** $f(x) = (1-p)^{x-1}p$, $x = 1, 2, \ldots$

**Poisson** $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, 2, \ldots$

**Uniform** $f(x) = \frac{1}{m}$, $x = 1, 2, \ldots, m$

**Negative Binomial (Optional)** $f(x) = \binom{x-1}{r-1}p^r(1-p)^{x-r}$, $x = r, r+1, r+2, \ldots$

**Hypergeometric (Optional)** $f(x) = \frac{\binom{N}{x} \binom{M}{n-x}}{\binom{N+M}{n}}$, $0 \leq x \leq n$, $x \leq N$, $n-x \leq M$
8. (20%) Let $X$ be a continuous random variable (r.v.) having the probability density function (p.d.f.) $f(x)$, then the mean $\mu$, variance $\sigma^2$, and the corresponding moment-generating function $\phi(t)$ are defined as follows.

$$\mu = E[X] = \int_{-\infty}^{\infty} [xf(x)]\,dx$$

$$\sigma^2 = Var(X) = E[(X - \mu)^2] = E[X^2] - (E[X])^2$$

$$\phi(t) = E[e^{tX}] = \int_{-\infty}^{\infty} [e^{tx}f(x)]\,dx$$

Moreover, we know that

$$\mu = \phi'(0), \text{ and } \sigma^2 = \phi''(0) - [\phi'(0)]^2$$

For a continuous type of r.v. $X$ which has one of the following probability density functions, derive the formula for the moment-generating function, and compute the mean and variance, respectively.

**Uniform $U(a, b)$** $f(x) = \frac{1}{b-a}, a \leq x \leq b$

**Exponential** $f(x) = \frac{1}{\theta}e^{-x/\theta}, 0 < x < \infty$

**Gamma** $f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha}x^{\alpha-1}e^{-x/\theta}, 0 < x < \infty$

**$\chi^2(r)$ Chi-Square** $f(x) = \frac{1}{\Gamma(r/2)2^{r/2}\pi}x^{(r/2)-1}e^{-x/2}, 0 < x < \infty$

**$N(\mu, \sigma^2)$ Normal** $f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}, -\infty < x < \infty$

**Beta (Optional)** $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1$