1. Write down the following probability density functions and compute their moment generating functions.

   (a) Binomial distribution with mean 30 and variance 12.
   (b) Poisson distribution with variance 4.
   (c) Exponential distribution with variance 4.
   (d) Normal distribution with mean 3, variance 4.
   (e) \( \chi^2 \) distribution with the degrees of freedom 12.

2. Let a r.v. \( X \) have the probability density function \( f(x) = \frac{x}{2}\sin(\pi x), 0 \leq x \leq 1 \).

   (a) Find the mean \( \text{E}(X) \) and \( \text{Var}(X) \).
   (b) Find the c.d.f. \( F(x) = P(X \leq x) \).
   (c) Find the 25th percentile.
   (d) Find the median.
   (e) Find the 3rd quartile.
3. A box contains five marbles numbered 1 through 5. The marbles are selected one at a time without replacement. A match occurs if marble numbered \( k \) is the \( k \)th marble selected. Let the event \( A_i \), denote a match on the \( i \)th draw, \( 1 \leq i \leq 5 \).

(a) Find \( P(A_i) \) for \( 1 \leq i \leq 5 \).

(b) Find \( P(A_i \cap A_j) \), where \( 1 \leq i < j \leq 5 \).

(c) Find \( P(A_i \cap A_j \cap A_k) \), where \( 1 \leq i < j < k \leq 5 \).

(d) Find \( P(A_i \cap A_j \cap A_k \cap A_m) \), where \( 1 \leq i < j < k < m \leq 5 \).

(e) Find \( P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) \).

4. Let \( X \) have a logistic distribution with p.d.f.

\[
f(x) = \frac{e^{-x}}{(1 + e^{-x})^2}, \quad -\infty < x < \infty
\]

Show that \( Y = 1/(1 + e^{-X}) \sim U(0, 1) \)
5. Suppose that 2000 points are independently and randomly selected from the unit square 
\( S = \{(x, y) : 0 \leq x, y \leq 1\} \). Let \( Y \) equal the number of points that fall in 
\( R = \{(x, y) : |x + y| \leq 1 \text{ and } |x - y| \leq 1\} \).

(a) How is \( Y \) distributed?
(b) Give the mean and variance of \( Y \).
(c) What is the expected value of \( Y/500 \)?
(d) What is \( P(Y \leq 100) \)?

6. If the moment-generating function of \( X \) is \( M(t) = (1 - 2t)^{-12}, t < 1/2 \).

Find \( \operatorname{E}(X) \) and \( \operatorname{Var}(X) \).
7. Let $X$ have the p.d.f. $f(x) = \theta x^{\theta-1}, \ 0 < x < 1, 0 < \theta < \infty$, and let $Y = -2\theta \ln X$.

(a) What is the moment-generating function of $Y$?
(b) How is $Y$ distributed?

8. Let $X_1 \sim b(n_1, p)$ and $X_2 \sim b(n_2, p)$ be independent r.v.’s. Define $Y = X_1 + X_2$.

(a) What is $M_Y(t)$?
(b) How is $Y$ distributed?
9. Show that the sum of $n$ independent Poisson random variables with respective means $\lambda_1, \lambda_2, \ldots, \lambda_n$ is Poisson with mean $\lambda = \sum_{i=1}^{n} \lambda_i$.

10. Let $Z_i \sim N(0, 1)$, for $1 \leq i \leq n$ and define $W = \sum_{i=1}^{n} Z_i^2$.

   (a) Find the moment-generating function for $Z_1$.
   (b) Find the moment-generating function for $W$.
   (c) How is $W$ distributed?