# The Weibull Distribution

The Weibull distribution is widely used for modeling the lifetimes of an *electronic* component (*device*). The nonnegative random variable $X$ with distribution function

$$F(x) = 1 - e^{-(x/\beta)^\alpha}, \quad x > 0$$
$$= 0, \quad x \leq 0$$

is said to have a Weibull distribution with *shape parameter* $\alpha > 0$ and *scale parameter* $\beta > 0$. The p.d.f. of the Weibull distribution is

$$f(x) = F'(x) = \tfrac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}, \quad x > 0$$
$$= 0, \quad x \leq 0$$

Note that the exponential distribution is a special case of Weibull distribution with $\alpha = 1$.

Now the expectation and variance of a Weibull distribution could be copmputed by

$$E(X) = \int_0^\infty \tfrac{\alpha}{\beta^\alpha} t^\alpha e^{-(x/\beta)^\alpha} dx = \beta\Gamma(1 + \tfrac{1}{\alpha})$$
$$E(X^2) = \int_0^\infty \tfrac{\alpha}{\beta^\alpha} t^{\alpha+1} e^{-(x/\beta)^\alpha} dx = \beta^2\Gamma(1 + \tfrac{2}{\alpha})$$
$$Var(X) = E(X^2) - (E(X))^2 = \beta^2 \left(\Gamma(1 + \tfrac{2}{\alpha}) - [\Gamma(1 + \tfrac{1}{\alpha})]^2\right)$$

**Example**: The lifetime, measured in years, of a brand of mobile device has a Weibull distribution with parameters $\alpha = 2$ and $\beta = 13$. Compute the probability of a mobile device fails before the expiration of a two-year warranty.

**Solution**: $P(X \leq 2) = 1 - exp[-(2/13)^2] \approx 0.0234$.

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X=0.1:0.2:18;
a=2; b=4;  Y1=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
a=2; b=7;  Y2=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
a=2; b=10; Y3=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
a=2; b=13; Y4=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
plot(X,Y1,'m-',X,Y2,'g-',X,Y3,'b-',X,Y4,'r-');
legend('Weibull(2,4)','Weibull(2,7)','Weibull(2,10)','Weibull(2,13)')
title('Weibull(a,b): f(x)=(a/b^a)x^{a-1}exp[-(x/b)^a]')
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