Partial Solutions for Assignment 2

(1) Let the random variable X have a p.m.f.

\[ f(x) = \frac{(|x| + 1)^2}{9}, \quad x = -1, 0, 1 \]

Then (a) \( E(X) = 0 \), (b) \( E(X^2) = \frac{8}{9} \), and (c) \( E(3X^2 - 2X + 4) = \frac{20}{3} \).

(2) Let \( X \) have a Poisson distribution of variance 4. Then (a) \( P(X = 4) = (e^{-4}4^4)/(4!) \approx 0.1954 \) and (b) \( P(2 \leq X \leq 6) \approx 0.889 - 0.092 = 0.797 \).

(3) It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let \( X \) equal the number with no health insurance in a random sample of \( n = 15 \) Americans.

(a) \( X \sim b(15, 0.2) \).
(b) \( E(X) = 3, Var(X) = 2.4, \) and \( Std. = \sqrt{2.4} = 1.5492 \).
(c) \( P(X \geq 2) \approx 1 - P(X = 0) - P(X = 1) = 0.8329 \).

(4) Let \( W \) have a geometric distribution with parameter \( p \).

(a) \( P(W = w) = (1 - p)^{w-1}p, \ w = 1, 2, \ldots \).
(b) \( E[X] = 1/p, Var(X) = (1 - p)/p^2 \).
(c) \( P(W > k) = \sum_{i=k+1}^{\infty} (1 - p)^{i-1}p = (1 - p)^k \). Thus \( P(W > (k+j)|W > k) = \frac{(1-p)^{k+j}}{(1-p)^k} = P(W > j) \). where \( k, j \) are nonnegative integers.

(5) See Lecture Notes.

(6) The probability mass functions and moment-generating functions are as follows.

(a) \( E(X) = \frac{1}{p}, \) then \( p = 0.8, M(t) = \frac{0.8e^t}{1-0.2e^t} \).
(b) \( X \sim b(50, 0.6), M(t) = (0.4 + 0.6e^t)^{50} \).
(c) \( f(x) = \frac{e^{-4}4^x}{x!}, \ x = 0, 1, \ldots \), \( M(t) = e^{4(e^t-1)} \).
(7) Implement the following Matlab codes and print out the results.

```matlab
\%  
\% Script file: h2p7.m - Discrete Distributions  
\%  
subplot(2,2,1)  
X=1:10; Y=geopdf(X,0.5); bar(X,Y,0.8);  
legend('Geometric Distribution: p=0.5',1)  
subplot(2,2,2)  
X=0:10; Y=binopdf(X,10,0.6); bar(X,Y,0.8)  
legend('X \sim b(10,0.6), mode=6',1)  
subplot(2,2,3)  
X=0:10; Y=poisspdf(X,4); bar(X,Y,0.8)  
legend('Poisson Distribution: \lambda =4',1)  
subplot(2,2,4)  
X=0:11; Y=binopdf(X,11,0.5); bar(X,Y,0.8)  
legend('X \sim b(11,0.5), mode=5,6',2)
```

![Geometric Distribution: p=0.5](image1)
![X \sim b(10,0.6), mode=6](image2)
![Poisson Distribution: \lambda =4](image3)
![X \sim b(11,0.5), mode=5,6](image4)

Figure 1: H2.P7 Solution.
(8) Suppose that 2000 points are independently and randomly selected from the unit square $S = \{(x, y) : 0 \leq x, y \leq 1\}$. Let $Y$ equal the number of points that fall in $A = \{(x, y) : x^2 + y^2 \leq 1\}$.

(a) $b(2000, \frac{\pi}{4})$, i.e., binomial distribution
(b) $500\pi$, $500\pi(1-\frac{\pi}{4})$
(c) \(\pi\)
(d) $\sum_{k=0}^{100} C(2000, k)(\pi/4)^k(1 - \pi/4)^{2000-k}$

(9) The probability mass (density) function (p.m.f. or p.d.f.) and the space of the random variable for the problems are given as follows.

(a) $X \sim b(50, 0.8)$, $f(x) = \binom{50}{x} (0.8)^x (0.2)^{50-x}, 0 \leq x \leq 50$.
(b) $f(x) = (0.5)^x$, $x = 1, 2, 3\cdots$
(c) $f(x) = \frac{e^{-4x}}{x!}$, $x = 0, 1, 2,\cdots$
(d) $f(x) = \frac{1}{10}e^{-x/10}$, $x \geq 0$
(e) $f(x) = \frac{1}{16}x^2e^{-x/2}$, $x > 0$.
(f) $f(x) = \frac{1}{\sqrt{32\pi}}e^{-\frac{(x-5)^2}{32}}$, $-\infty < x < \infty$