Partial Solutions for Assignment 2

(1) Let the random variable X have a p.m.f.

$$f(x) = \frac{(|x|+1)^2}{9}, \quad x = -1,\ 0,\ 1$$

Then (a) $E(X) = 0$, (b) $E(X^2) = \frac{8}{9}$, and (c) $E(3X^2 - 2X + 4) = \frac{20}{3}$.

(2) Let X have a Poisson distribution of variance 4. Then (a) $P(X = 4) = (e^{-4}4^4)/(4!) \approx 0.1954$ and (b) $P(2 \le X \le 6) \approx 0.889 - 0.092 = 0.797$

(3) It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let X equal the number with no health insurance in a random sample of $n = 15$ Americans.

(a) $X \sim b(15, 0.2)$.

(b) $E(X) = 3$, $Var(X) = 2.4$, and $Std. = \sqrt{2.4} = 1.5492$.

(c) $P(X \ge 2)1 - P(X = 0) - P(X = 1) = 0.8329$.

(4) Let W have a geometric distribution with parameter p.

(a) $P(W = w) = (1-p)^{w-1}p, \quad w = 1, 2, \cdots$.

(b) $E[X] = 1/p$, $Var(X) = (1-p)/p^2$.

(c) $P(W > k) = \sum_{i=k+1}^{\infty}(1-p)^{i-1}p = (1-p)^k$. Thus $P(W > (k+j)|W > k) = \frac{(1-p)^{k+j}}{(1-p)^k} = P(W > j)$. where k, j are nonnegative integers.

(5) See *Lecture Notes.*

(6) The probability mass functions and moment-generating functions are as follows.

(a) $E(X) = \frac{1}{p}$, then $p = 0.8$, $M(t) = \frac{0.8e^t}{1-0.2e^t}$.

(b) $X \sim b(50, 0.6)$, $M(t) = (0.4 + 0.6e^t)^{50}$.

(c) $f(x) = \frac{e^{-4}4^x}{x!}$, $x = 0, 1, \cdots$, $M(t) = e^{4(e^t - 1)}$.

(7) Implement the following Matlab codes and print out the results.

```
%
% Script file: h2p7.m - Discrete Distributions
%
subplot(2,2,1)
X=1:10; Y=geopdf(X,0.5); bar(X,Y,0.8);
legend('Geometric Distribution: p=0.5',1)
subplot(2,2,2)
X=0:10; Y=binopdf(X,10,0.6); bar(X,Y,0.8)
legend('X \sim b(10,0.6), mode=6',1)
subplot(2,2,3)
X=0:10; Y=poisspdf(X,4); bar(X,Y,0.8)
legend('Poisson Distribution: \lambda =4',1)
subplot(2,2,4)
X=0:11; Y=binopdf(X,11,0.5); bar(X,Y,0.8)
legend('X \sim b(11,0.5), mode=5,6',2)
```

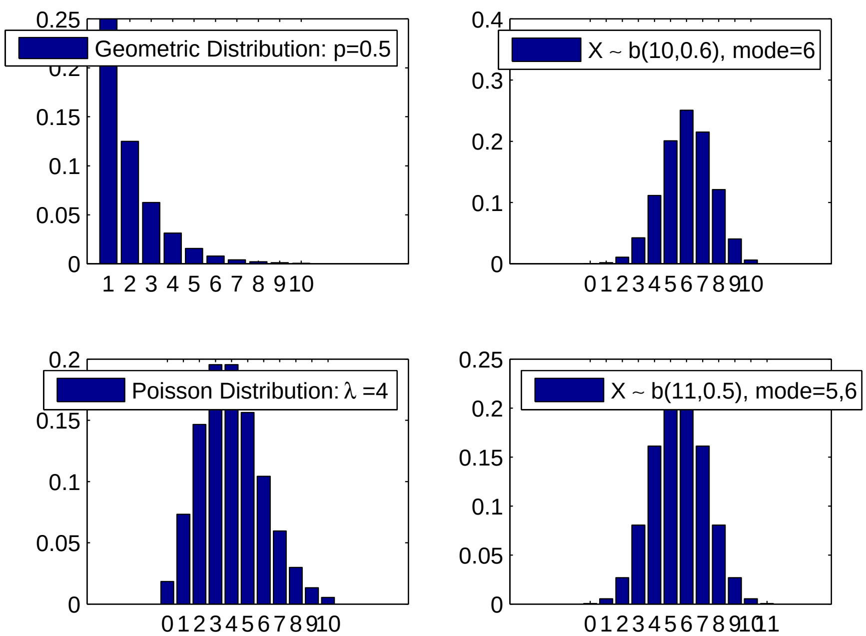

Figure 1: H2.P7 Solution.

(8) Suppose that 2000 points are independently and randomly selected from the unit square $S = \{(x, y) : 0 \le x, y \le 1\}$. Let Y equal the number of points that fall in $A = \{(x, y) : x^2 + y^2 \le 1\}$.

(a) $b(2000, \frac{\pi}{4})$, i.e., binomial distribution

(b) 500π, $500\pi(1\text{-}\frac{\pi}{4})$

(c) π

(d) $\sum_{k=0}^{k=100} C(2000, k)(\pi/4)^k(1 - \pi/4)^{2000-k}$

(9) The probability mass (density) function (p.m.f. or p.d.f.) and the space of the random variable for the problems are given as follows.

(a) $X \sim b(50, 0.8)$, $f(x) = \left(\begin{array}{c} 50 \\ x \end{array}\right)(0.8)^x(0.2)^{50-x}, \;\; 0 \le x \le 50.$

(b) $f(x) = (0.5)^x, \;\; x = 1, 2, 3 \cdots$

(c) $f(x) = \frac{e^{-4}4^x}{x!}, \;\; x = 0, 1, 2, \cdots$

(d) $f(x) = \frac{1}{10}e^{-x/10}, \;\; x \ge 0$

(e) $f(x) = \frac{1}{16}x^2 e^{-x/2}, \;\; x > 0.$

(f) $f(x) = \frac{1}{\sqrt{32\pi}}e^{-(x-5)^2/32}, \quad -\infty < x < \infty$