Midterm Exam for CS3332(01), Spring 2015

$Name:$ ______________ $SN:$ ______________ $Gn:$ _____ $Index:$ _____

(30pts) 1. Choose the *best* (unique) solution for each of the following problems.

(1)**(a)** Let X be a gamma distribution with p.d.f. $f(x) = \frac{1}{16}x^2e^{-x/2},\ 0 \le x \le \infty$, then the moment-generating function is

(1) $\frac{1}{(1-2t)^3}$, (2) $\frac{1}{(1-3t)^2}$, (3) $\frac{1}{1-2t}$, (4) $\frac{1}{1-3t}$, (5) none

(2)**(b)** Let X have a density function $f(x) = \frac{1}{2}e^{-x/2},\ x \ge 0$, then the variance of X is

(1) 2, (2) 4, (3) $\frac{1}{2}$, (4) $\frac{1}{4}$, (5) none

(3)**(c)** Let X have a density function $f(x) = \frac{1}{2}e^{-x/2},\ x \ge 0$, then the median of X is

(1) e^{-1}, (2) e^{-2}, (3) $2ln2$, (4) $(ln2)/2$, (5) none

(3)**(d)** Let X be a gamma distribution with p.d.f. $f(x) = \frac{1}{16}x^2e^{-x/2},\ 0 \le x \le \infty$, then the variance of X is

(1) 3, (2) 6, (3) 12, (4) 16, (5) none

(4)**(e)** Let X be a χ^2 distribution whose moment-generating function $\phi(t) = \frac{1}{(1-2t)^2},\ t < \frac{1}{2}$, then the variance Var(X) is

(1) 1, (2) 2, (3) 4, (4) 8, (5) none

(1)**(f)** Let X be a χ^2 distribution whose moment-generating function $\phi(t) = \frac{1}{(1-2t)},\ t < \frac{1}{2}$, then the first quartile of X is

(1) $2\ln(\frac{4}{3})$, (2) $2\ln(\frac{2}{3})$, (3) 0.25, (4) 0.75, (5) none

(2)**(g)** Let $X \sim N(3,4)$, then the moment-generating function of X is

(1) e^{3t+4t^2}, (2) e^{3t+2t^2}, (3) $\frac{1}{4-3e^t}$, (4) $\frac{1}{3-4e^t}$, (5) none

(3)**(h)** Let $X \sim N(3,4)$, then the median of X is

(1) 1, (2) 2, (3) 3, (4) 4, (5) none

(4)**(i)** Let $Z \sim N(0,1)$ and define $X = 2Z + 3$, then $Var(X)$ equals

(1) 1, (2) 2, (3) 3, (4) 4, (5) none

(2)**(j)** Let $X \sim N(3,4)$ and define $Y = (X-3)/2$, then $Var(Y^2)$ equals

(1) 1, (2) 2, (3) 3, (4) 4, (5) none

(40pts)2. Fill the following blanks.

(a) Let A and B be independent events with $P(A) = 0.7$ and $P(B) = 0.2$, then

$P(A \cap B) =$ __0.14__

$P(A \cup B) =$ __0.76__

(b) Let $X \sim b(50, 0.6)$, then

$E(X) =$ __30__

$Var(X) =$ __12__

$M(t) =$ __$(0.4 + 0.6e^t)^{50}$__

(c) If the moment-generating function of X is $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$, then

$E(X) =$ __2__

$Var(X) =$ __$\frac{4}{5}$__

(d) A random variable X has p.d.f. $f(x) = \frac{x+1}{2}, \quad -1 < x < 1$. Then

the median of $X =$ __$-1 + \sqrt{2}$__

(e) Let $X \sim N(3, 4)$, then

the p.d.f. of X is $f(x) =$ __$\frac{1}{\sqrt{8\pi}} e^{-(x-3)^2/8}$__

the moment-generating function $\phi_X(t) =$ __e^{3t+2t^2}__

(f) Let a random variable X have the Poisson distribution with variance 4.

$E(X) =$ __4__

$M(t) =$ __$e^{4(e^t-1)}$__

(g) Let a random variable X have the geometric distribution with $E(X) = 4$. Then

$Var(X) =$ __12__

$M(t) =$ __$\frac{e^t}{4-3e^t}$__

(h) Let a random variable X have an exponential distribution with $Var(X) = 4$. Then

$E(X) =$ __2__

The moment-generating function $\phi_X(t) =$ __$\frac{1}{1-2t}$__

(10pts)3. A box contains four marbles numbered 1 through 4. The marbles are selected one at a time without replacement. A match occurs if marble numbered k is the kth marble selected. Let the event A_i, denote a match on the ith draw, i=1,2,3,4.

(a) Find P(A_i), for i=1,2,3,4.

(b) Find P($A_i \cap A_j$), where $1 \leq i < j \leq 4$.

(c) Find P($A_i \cap A_j \cap A_k$), where $1 \leq i < j < k \leq 4$.

(d) Find P($A_1 \cup A_2 \cup A_3 \cup A_4$).

See Lecture Notes

(10pts)4. Let the random variable X have a negative binomial distribution whose probability mass function (p.m.f.) is

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \quad \forall\, x = r,\ r+1,\ r+2,\ \cdots$$

(a) Show your procedure to find the moment-generating function $M(t)$.

(b) Show your procedure to find the expectation $E(X)$.

(c) Compute the variance $Var(X)$.

Hint: $(1-\omega)^{-r} = \sum_{k=0}^{\infty} \binom{k+r-1}{r-1} \omega^k$

See Lecture Notes

(10pts)5. Write Simple Matlab Codes to plot each of the following probability mass functions with a given random variable X.

(a) $X \sim b(10, 0.8)$.

(b) X has a geometric distribution with the expectation $E(X) = 4$.

(c) X has a Poisson distribution with variance 4.

See Homework Assignment