Midterm Exam for CS3332(01), Spring 2015

Name : ______________ *SN* : ______________ *Gn* : _____ *Index* : _____

(30pts) 1. Choose the *best* (unique) solution for each of the following problems.

( )**(a)** Let X be a gamma distribution with p.d.f. $f(x) = \frac{1}{16}x^2e^{-x/2},\ 0 \leq x \leq \infty$, then the moment-generating function is

(1) $\frac{1}{(1-2t)^3}$, (2) $\frac{1}{(1-3t)^2}$, (3) $\frac{1}{1-2t}$, (4) $\frac{1}{1-3t}$, (5) none

( )**(b)** Let X have a density function $f(x) = \frac{1}{2}e^{-x/2},\ x \geq 0$, then the variance of X is

(1) 2, (2) 4, (3) $\frac{1}{2}$, (4) $\frac{1}{4}$, (5) none

( )**(c)** Let X have a density function $f(x) = \frac{1}{2}e^{-x/2},\ x \geq 0$, then the median of X is

(1) e^{-1}, (2) e^{-2}, (3) $2ln2$, (4) $(ln2)/2$, (5) none

( )**(d)** Let X be a gamma distribution with p.d.f. $f(x) = \frac{1}{16}x^2e^{-x/2},\ 0 \leq x \leq \infty$, then the variance of X is

(1) 3, (2) 6, (3) 12, (4) 16, (5) none

( )**(e)** Let X be a χ^2 distribution whose moment-generating function $\phi(t) = \frac{1}{(1-2t)^2},\ t < \frac{1}{2}$, then the variance Var(X) is

(1) 1, (2) 2, (3) 4, (4) 8, (5) none

( )**(f)** Let X be a χ^2 distribution whose moment-generating function $\phi(t) = \frac{1}{(1-2t)},\ t < \frac{1}{2}$, then the first quartile of X is

(1) $2\ln(\frac{4}{3})$, (2) $2\ln(\frac{2}{3})$, (3) 0.25, (4) 0.75, (5) none

( )**(g)** Let $X \sim N(3,4)$, then the moment-generating function of X is

(1) e^{3t+4t^2}, (2) e^{3t+2t^2}, (3) $\frac{1}{4-3e^t}$, (4) $\frac{1}{3-4e^t}$, (5) none

( )**(h)** Let $X \sim N(3,4)$, then the median of X is

(1) 1, (2) 2, (3) 3, (4) 4, (5) none

( )**(i)** Let $Z \sim N(0,1)$ and define $X = 2Z + 3$, then $Var(X)$ equals

(1) 1, (2) 2, (3) 3, (4) 4, (5) none

( ) **(j)** Let $X \sim N(3,4)$ and define $Y = (X-3)/2$, then $Var(Y^2)$ equals

(1) 1, (2) 2, (3) 3, (4) 4, (5) none

bf Answer:

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5, 4, 3, 2, 1;  5, 4, 3, 2, 1
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(40pts)2. Fill the following blanks.

(a) Let A and B be independent events with $P(A) = 0.7$ and $P(B) = 0.2$, then

$P(A \cap B) =$ ________________

$P(A \cup B) =$ ________________

(b) Let $X \sim b(50, 0.6)$, then

$E(X) =$ ____________

$Var(X) =$ ____________

$M(t) =$ ________________________

(c) If the moment-generating function of X is $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$, then

$E(X) =$ ____________

$Var(X) =$ ____________

(d) A random variable X has p.d.f. $f(x) = \frac{x+1}{2}, \quad -1 < x < 1$. Then

$the\ median\ of\ X =$ ________________________

(e) Let $X \sim N(3, 4)$, then

the p.d.f. of X is $f(x) =$ ____________________________,

the moment-generating function $\phi_X(t) =$ ________________________

(f) Let a random variable X have the Poisson distribution with variance 4.

$E(X) =$ ____________

$M(t) =$ ________________________

(g) Let a random variable X have the geometric distribution with $E(X) = 4$. Then

$Var(X) =$ ____________

$M(t) =$ ________________________

(h) Let a random variable X have an exponential distribution with $Var(X) = 4$. Then

$E(X) =$ ____________

The moment-generating function $\phi_X(t) =$ ________________________

(10pts)3. A box contains four marbles numbered 1 through 4. The marbles are selected one at a time without replacement. A match occurs if marble numbered k is the kth marble selected. Let the event A_i, denote a match on the ith draw, i=1,2,3,4.

(a) Find $P(A_i)$, for i=1,2,3,4.

(b) Find $P(A_i \cap A_j)$, where $1 \leq i < j \leq 4$.

(c) Find $P(A_i \cap A_j \cap A_k)$, where $1 \leq i < j < k \leq 4$.

(d) Find $P(A_1 \cup A_2 \cup A_3 \cup A_4)$.

(10pts)4. Let the random variable X have a negative binomial distribution whose probability mass function (p.m.f.) is

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \quad \forall\, x = r,\ r+1,\ r+2,\ \cdots$$

(a) Show your procedure to find the moment-generating function $M(t)$.

(b) Show your procedure to find the expectation $E(X)$.

(c) Compute the variance $Var(X)$.

Hint: $(1-\omega)^{-r} = \sum_{k=0}^{\infty} \binom{k+r-1}{r-1} \omega^k$

(10pts)5. Write Simple Matlab Codes to plot each of the following probability mass functions with a given random variable X.

(a) $X \sim b(10, 0.8)$.

(b) X has a geometric distribution with the expectation $E(X) = 4$.

(c) X has a Poisson distribution with variance 4.