Assignment 4: Sampling Distributions

Due in class on June 10, 2015

(1) Let X_1 and X_2 be two independent random variables, each with the same probability distribution given as follows.

$$f(x) = \frac{1}{2}e^{-x/2}, \quad x \geq 0$$

(a) Compute the probability distribution function of the new random variable $Y = X_1 + X_2$.

(b) What type of probability distribution is your answer in **(a)**?

(2) Let the moment-generating function of X be

$$\phi(t) = e^{3t+2t^2}, \quad -\infty < t < \infty$$

(a) Find the mean E(X) and variance Var(X).

(b) Name the distribution of X.

(c) Write down the p.d.f $f(x)$ for X.

(3) Let $X_1 \sim b(n_1, p)$ and $X_2 \sim b(n_2, p)$ be independent r.v.'s. Define $Y = X_1 + X_2$.

(a) What is $M_Y(t)$?

(b) How is Y distributed?

(4) If $X \sim N(\mu, \sigma^2)$, show that $Y = (aX + b) \sim N(a\mu + b, a^2\sigma^2), \;\; a \neq 0$.

(5) The joint probability density function for two continuous random variables X and Y is given as follows.

$$f(x, y) = \begin{cases} \beta xy & 0 < x < 2 \;\; and \;\; 0 < y < 4 \\ 0 & elsewhere \end{cases}$$

(a) Determine the constant β to make it a joint p.d.f.

(b) Determine the conditional probability distribution function $f(x|y)$.

(c) Compute the covariance between X and Y.

(6) Show that the sum of n independent Poisson random variables with respective means $\lambda_1, \lambda_2, \ldots, \lambda_n$ is Poisson with mean $\lambda = \sum_{i=1}^{n} \lambda_i$.

(7) Let $Z_i \sim N(0,1)$, for $1 \le i \le 7$ and define $W = \sum_{i=1}^{7} Z_i^2$. Find P(1.69< W <14.07).

(8) Assume there are 100 obervations, denoted by x_i, $1 \le i \le 100$, and each is drawn from a population with a continuous distribution in [0,2].

(a) Give the formula for the sample mean $\overline{X}$ and sample variance S^2 in this problem.

(b) Compute the sample mean and sample variance for $\overline{X}$.

(c) Try to give an approximate distribution for $\overline{X}$ as best as you can and explain your reason.

(d) Compute the probability that the sample mean value is larger than 1.02. Note that you can express your solution with the cumulative standard normal distribution function

$$\Phi(r) = \int_{-\infty}^{r} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

(10) Let $X_1, X_2, \ldots, X_{30}$ be a random sample of size 30 from a Poisson distribution with a mean 2/3. Approximate

(a) $P(15 < \sum_{i=1}^{30} X_i \le 22)$.

(b) $P(21 \le \sum_{i=1}^{30} X_i < 27)$.

(11) Let $X_1, X_2, \cdots, X_n$ be a random sample of an exponential distribution with mean θ, that is, $f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0$.

(a) Show that $P(min(X_1, X_2, \cdots, X_n) > 2) = e^{-2n/\theta}$.

(b) Show that $P(max(X_1, X_2, \cdots, X_n) > 2) = 1 - (1 - e^{-2/\theta})^n$.

(c) Find a minimum n such that $P(min(X_1, X_2, \cdots, X_n) > 2) \le 3\%$ when $\theta = 2$.

(d) Find a minimum n such that $P(max(X_1, X_2, \cdots, X_n) > 2) \ge 90\%$ when $\theta = 2$.

(12) Let $W_1 < W_2 < \cdots < W_n$ be the order statistics of a random sample of zise n from the uniform distribution U(0,1).

(a) Find the probability density function of W_1.

(b) Find the probability density function of W_n.

(c) Use the result of **(a)** to find $E(W_1)$.

(d) Use the result of **(b)** to find $E(W_n)$.