Assignment 3: Continuous Distributions

Due in class of May 8, 2015

(1) Let a r.v. X have the probability density function $f(x) = \frac{1}{2}sin(x)$, $0 \leq x \leq \pi$.

(a) Find the expectation E(X) and variance Var(X).

(b) Sketch the graph of the p.d.f. of X.

(c) Sketch the graph of the distribution function of X.

(2) Let X have the p.d.f. $f(x) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$, and let $Y = -2\theta lnX$.

(a) How is Y distributed? (write down the p.d.f. of Y).

(b) What is the moment-generating function of Y?

(3) Write down the following probability density functions and *derive* their moment generating functions.

(a) Exponential distribution with variance 4.

(b) Normal distribution with mean 3, variance 4.

(c) χ^2 distribution with the degrees of freedom 12.

(4) Plot the following exponential density functions in a single frame.

(a) An exponential function with mean 1.

(b) An exponential function with mean 2.

(c) An exponential function with mean 4.

(d) An exponential function with mean 7.

(5) Plot the following $\chi^2(r)$ density functions in a single frame.

(a) $\chi^2(1)$.

(b) $\chi^2(2)$.

(c) $\chi^2(4)$.

(d) $\chi^2(7)$.

(6) Plot the following normal density functions in a single frame.

(a) $X \sim N(0, 1^2)$

(b) $X \sim N(0, 2^2)$

(c) $X \sim N(0, (2.5)^2)$

(d) $X \sim N(0, 3^2)$

(7) Let X have a logistic distribution with the p.d.f.

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2}, \quad -\infty < x < \infty$$

Show that $Y = \frac{1}{1+e^{-X}}$ has a U(0,1) uniform distribution.

(8) Let X have an exponential distribution with a mean of $\theta = 20$. Compute

(a) P$(10 < X < 30)$

(b) P$(X > 30)$

(c) P$(X > 40 | X > 10)$

(9) The p.d.f. of time X to failure of an electronic component is

$$f(x) = \frac{2x}{10^6} e^{-(x/1000)^2}, \quad 0 < x < \infty$$

(a) Computer $P(X > 2000)$.

(b) Determine the *75th* percentile, $\pi_{0.75}$, of the distribution.

(c) Find the *10th* and *60th* percentiles, $\pi_{0.10}$, $\pi_{0.60}$,

(10*) Let X, Y be a random sample of size 2 from $\sim N(3, 0.25)$. Define $Z = 2(X-3)$, $U = 2(Y-3)$, $W = Z^2$, $V = Z + U$.

(a) Write down the probability density function of X.

(b) Show that Z has the standard normal distribution.

(c) What is the moment-generating function of Z?

(d) Show that $W \sim \chi^2(1)$.

(e) What is the moment-generating function of W?

(f) What is the moment-generating function of V?

(g) How is V distributed?

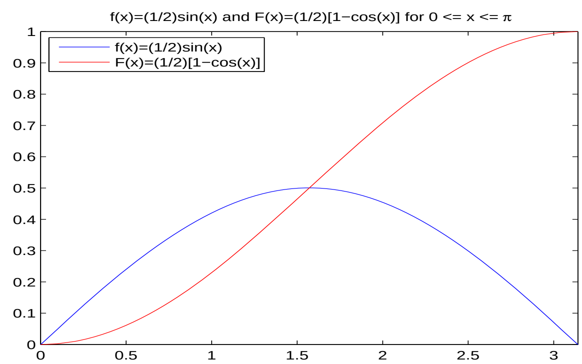
(h) What is the probability density function of V?

Partial Solutions for h3/2014S

(1)(a) $E(X) = \int_0^\pi \frac{x}{2}\sin(x)dx = \pi/2$, $Var(X) = \frac{\pi^2}{4} - 2$

(1)(b)(c)

```
% Script file: h3p1.m - Problem 1(b)(c) of H3
% Plots of f(x)=(1/2)sin(x) and F(x)=(1/2)[1-cos(x)],  0<=x<=\pi
%
X=0:(pi/32):pi;
f=0.5*sin(X);
F=0.5*(1-cos(X));
plot(X,f,'b-',X,F,'r-');
axis([0,pi, 0,1]);
legend('f(x)=(1/2)sin(x)','F(x)=(1/2)[1-cos(x)]',2);
title('f(x)=(1/2)sin(x) and F(x)=(1/2)[1-cos(x)] for 0 <= x <= \pi')
```



(2) Let X have the p.d.f. $f(x) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$, and let $Y = -2\theta lnX$.

(a) $P(Y \leq y) = P(X \geq e^{-y/2\theta}) = \int_{e^{-y/2\theta}}^1 \theta x^{\theta-1}dx = 1 - e^{-y/2}$, $y > 0$, then $f(y) = \frac{1}{2}e^{-y/2}$, $y > 0$, thus, Y has an exponential distribution with mean 2.

(b) $M_Y(t) = \frac{1}{1-2t}$, $t < \frac{1}{2}$.

(3) Write down the following probability density functions and *derive* their moment generating functions.

(a) *Exponential distribution with variance 4* ($\phi(t) = \frac{1}{1-\mathbf{2}t}$).

(b) *Normal distribution with mean 3, variance 4* ($\phi(t) = e^{3t+2t^2}$).

(c) χ^2 *distribution with the degrees of freedom 12* ($\phi(t) = \frac{1}{(1-2t)^6}$).