Partial Solutions for Assignment 2

(1) Let the random variable $X$ have a p.d.f.

$$f(x) = \frac{(|x| + 1)^2}{9}, \ x = -1, \ 0, \ 1$$

Then (a) $E(X) = 0$, (b) $E(X^2) = \frac{8}{9}$, and (c) $E(3X^2 - 2X + 4) = \frac{20}{3}$.

(2) Let $X$ have a Poisson distribution of variance 4. Then (a) $P(X = 4) = (e^{-4}4^4)/(4!) \approx 0.1954$ and (b) $P(2 \leq X \leq 6) \approx 0.889 - 0.092 = 0.797$.

(3) It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let $X$ equal the number with no health insurance in a random sample of $n = 15$ Americans.

(a) $X \sim b(15, 0.2)$.
(b) $E(X) = 3, Var(X) = 2.4$, and $Std. = \sqrt{2.4} = 1.5492$.
(c) $P(X \geq 2)1 - P(X = 0) - P(X = 1) = 0.8329$.

(4) Let $W$ have a geometric distribution with parameter $p$.

(a) $P(W = w) = (1 - p)^{w-1}p, \ w = 1, 2, \ldots$.
(b) $E[X] = 1/p, Var(X) = (1 - p)/p^2$.
(c) $P(W > k) = \sum_{i=k+1}^{\infty} (1 - p)^{i-1}p = (1 - p)^k$. Thus $P(W > (k + j)|W > k) = \frac{(1-p)^{k+j}}{(1-p)^k} = P(W > j)$, where $k, j$ are nonnegative integers.

(5) See Lecture Notes.

(6) The probability mass functions and moment-generating functions are as follows.

(a) $E(X) = \frac{1}{p}$, then $p = 0.8, M(t) = \frac{0.8e^t}{1-0.2e^t}$.
(b) $X \sim b(50, 0.6), M(t) = (0.4 + 0.6e^t)^{50}$.
(c) $f(x) = e^{-4\frac{x}{2}}x!, \ x = 0, 1, \ldots, M(t) = e^{4(e^t-1)}$. 
(7) Implement the following Matlab codes and print out the results.

```matlab
% Script file: h2p7.m - Discrete Distributions

subplot(2,2,1)
X=1:10; Y=geopdf(X,0.5); bar(X,Y,0.8);
legend('Geometric Distribution: p=0.5',1)

subplot(2,2,2)
X=0:10; Y=binopdf(X,10,0.6); bar(X,Y,0.8)
legend('X \sim b(10,0.6), mode=6',1)

subplot(2,2,3)
X=0:10; Y=poisspdf(X,4); bar(X,Y,0.8)
legend('Poisson Distribution: \lambda =4',1)

subplot(2,2,4)
X=0:11; Y=binopdf(X,11,0.5); bar(X,Y,0.8)
legend('X \sim b(11,0.5), mode=5,6',2)
```

Figure 1: H2.P7 Solution.
Suppose that 2000 points are independently and randomly selected from the unit square \( S = \{(x, y) : 0 \leq x, y \leq 1\} \). Let \( Y \) equal the number of points that fall in \( A = \{(x, y) : x^2 + y^2 \leq 1\} \).

(a) \( b(2000, \frac{\pi}{4}) \), i.e., binomial distribution
(b) \( 500\pi, 500\pi(1-\frac{\pi}{4}) \)
(c) \( \pi \)
(d) \( \sum_{k=0}^{100} C(2000, k)(\frac{\pi}{4})^k(1-\frac{\pi}{4})^{2000-k} \)

The probability mass (density) function and the space of the random variable for the problems are given as follows.

(a) \( X \sim b(50, 0.8), f(x) = \binom{50}{x} (0.8)^x (0.2)^{50-x}, \ 0 \leq x \leq 50. \)
(b) \( f(x) = (0.5)^x, \ x = 1, 2, 3 \cdots \)
(c) \( f(x) = \frac{e^{-4x}}{x!}, \ x = 0, 1, 2, \cdots \)
(d) \( f(x) = \frac{1}{10} e^{-x/10}, \ x \geq 0 \)
(e) \( f(x) = \frac{1}{16} x^2 e^{-x/2}, \ x > 0. \)
(f) \( f(x) = \frac{1}{32\pi} e^{-(x-5)^2/32}, \ -\infty < x < \infty \)