Solutions for CS3331: Test 3, Fall 2009

13:10-15:00, January 8, 2010

I. (50 pts) *Write Matlab codes to solve the following problems.*

(15 pts) 1. Using $sin(x) \approx \sum_{k=1}^{\infty} \frac{x^{2k-1}}{k!}$, we have

$$sin(1) = 1 - 1/6 + 1/120 + 1/5040 - 1/40320$$

(10 pts) 2.
```
H=hilb(4);
b=ones(4,1);
x=H\b
```

(10 pts) 3.
```
format short;
A=[-5, 2, 1; 2, -5, 2; 1, 2, 4];
P=poly(A)
X1=roots(P); X=sort(X1)
Y1=eig(A);   Y=sort(Y1)
d=norm(X-Y,2)
```

(15 pts) 4. `fzero(@(x)(x*exp(-x/2)+2*exp(-x/2)-0.5),5)`

II. (40 pts) *Answer the following questions.*

(10 pts) 5. The nonlinear systems of equations are listed below.

$$\begin{aligned} x^3 - 10x + y - z &= -3 \\ y^3 + 10y - 2x - 2z &= 5 \\ x + y - 10z + 2sin(z) &= -5 \end{aligned}$$

(5 pts) 6(a) $|f(x) - f(y)| = |\sqrt{x+2} - \sqrt{y+2}| = |\frac{(x+2)-(y+2)}{\sqrt{x+2}+\sqrt{y+2}}| < s|x-y|$, where $0 < s < 1$.

(5 pts) 6(b) The fixed point is 2, that is $f(2) = \sqrt{2+2} = 2$.

(20 pts) 7. Refer to the lecture notes.

III. (60 pts) *Answer the following questions.*

(20 pts) 8.

(a) $T_0(x) = 1,\ T_1(x) = x,\ T_2(x) = 2x^2 - 1,\ T_3(x) = 4x^3 - 3x,\ T_4(x) = 8x^4 - 8x^2 + 1.$

(b) easy

(c) $\langle T_0, T_0 \rangle = \pi$ and $\langle T_n, T_n \rangle = \frac{\pi}{2}$ for $n \geq 1$.

(d) $cos(\frac{\pi}{n}(k + \frac{1}{2})),\ n \in N,\ k \in Z$

(20 pts) 9.

(b) $0.25 \cdot \frac{(x-2.5)(x-5)}{(2-2.5)(2-5)} + 0.16 \cdot \frac{(x-2)(x-5)}{(2.5-2)(2.5-5)} + 0.04 \cdot \frac{(x-2)(x-2.5)}{(5-2)(5-2.5)}$

(c) $0.25 - 0.18 \cdot (x-2) + 0.044 \cdot (x-2)(x-2.5)$

(d) $0.044 \cdot x^2 - 0.378 \cdot x + 0.83$

(20 pts) 10. $(a, b, c, d) = (1, 2, 0, -1)$