Solutions for Test 1

1. $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$

2(a) $\|\mathbf{x}\|_1 = 10$ and $\|\mathbf{x}\|_\infty = 5$. **2(b)** $\|B\|_1 = 25$ and $\|B\|_\infty = 26$.

3.

$$H = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

4

$$(a)\ \ A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 6 \end{bmatrix} \qquad (b)\ \ B = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 6 \end{bmatrix} \qquad (c)\ \ x_{10} = 55$$

5(a) Decompose $A = LU$, where A is *unit lower* $- \Delta$, U is *upper* $- \Delta$.

5(b) Decompose $A = LU$, where A is *lower* $- \Delta$, U is *unit upper* $- \Delta$.

5(c) Let A be *positive definite*, decompose $A = LL^t$, where L is *lower* $- \Delta$.

6(a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 14 & 28 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$$

6(b) Use Gaussian Elimination with Partial Pivoting followed by Back Substitution to solve A**x**=**b**,

6(c) $x = [2, 1, -1]^t$

7(a)

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for i=2:n
  r=T(i,i-1)/T(i-1,i-1);
  T(i,i)=T(i,i)-r*T(i-1,i);
  T(i,i-1)=r;
end
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7(b) $3(n-1)$ *flops*

8 Show that $\|A\|_\infty \leq max_{1\leq i\leq m} \left\{\sum_{j=1}^{n} |a_{ij}|\right\}$ and show the equality holds for $\mathbf{x}$ with $x_j = sign(a_{Kj})$, where $\sum_{j=1}^{n} |a_{Kj}| = max_{1\leq i\leq m} \left\{\sum_{j=1}^{n} |a_{ij}|\right\}$