Assignment 4: Solving Nonlinear Equations

1. Solving nonlinear equations in one dimension and test your implementations by finding at least one root for each of the following equations. What termination criteria should you use? What convergence rate is achieved in each case? In summary, Let $f(x) = x^2 - x - 2$, $g(x) = e^{-x} - x$, and $h(x) = x + 4 + \frac{1}{3}sin(2x)$, find the zeros of f, g, h, respectively by

   (a) Newton's method with a *suitable* initial guess.

   (b) Functional iteration and fixed point theorem.

   (c) Matlab solutions by the script file $fzero.m$.

2. Find a solution of the following equations by Newton-Raphson method using $\mathbf{x}^{(0)} = [2, 1, 1]^t$ as an initial guess. What is the difference if we use $\mathbf{x}^{(0)} = [1, 2, 3]^t$ or other $\mathbf{x}^{(0)}$ as an initial guess?

$$
\begin{aligned}
xy \;-\; z^2 &= 1 \\
xyz \;-\; x^2 \;+\; y^2 &= 2 \\
e^x \;-\; e^y \;-\; z &= 3
\end{aligned}
$$

3. Solve the nonlinear systems of equations using

   (a) Newton's method with a *suitable* initial guess.

   (b) fixed-point iteration.

$$
\begin{aligned}
x^3 \;-\; 10x \;+\; y \;-\; z &= -3 \\
y^3 \;+\; 10y \;-\; 2x \;-\; 2z &= 5 \\
x \;+\; y \;-\; 10z \;+\; 2sin(z) &= -5
\end{aligned}
$$

4. Solve the nonlinear systems of equations using

   (a) Newton's method with a *suitable* initial guess.

   (b) fixed-point iteration.

$$
\begin{aligned}
x^2 \;+\; 20x \;+\; y^2 \;+\; z^2 &= 20 \\
x^2 \;+\; 20y \;+\; z^2 &= 20 \\
x^2 \;+\; y^2 \;-\; 40z &= 0
\end{aligned}
$$