Assignment 1: Mathematical Preliminaries and Computer Arithmetic

1. How many terms are required in the series

$$e = \sum_{k=0}^{\infty} \frac{1}{k!}$$

   to give e an error of at most $\frac{6}{10}$ unit in the 20th decimal place?

2. Consider the following Fibonacci sequence and its formula

$$x_{n+1} = x_n + x_{n-1} \quad for \ n \geq 1 \ \ with \ \ x_0 = x_1 = 1,$$

$$x_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^{n+1}$$

   Compute x_n, $41 \leq n \leq 47$ by both the recursive relation and the formula for each case.

3. Solve the following recurrence equations with initial value $x_1 = 1$.

   (a) $x_{n+1} - nx_n = 0$,   **(b)** $x_{n+1} - x_n = n$,   **(c)** $x_{n+1} - x_n = 2$.

4. When we write $\Pi_{i=1}^{n}(1 + \delta_i) = 1 + \epsilon$, where $|\delta_i| \leq 2^{-24}$, what is the range of possible values for ϵ?   Is $|\epsilon| \leq n \times 2^{-24}$ is a realistic bound?

5. In solving the quadratic equation $ax^2 + bx + c = 0$ by use of the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   there is a loss of significance when $4ac$ is small relative to b^2. Suggest a method to circumvent this difficulty and test the following cases.

Problem	a	b	c
(i)	1	-10^5	1
(ii)	1	-4	3.999999
(iii)	0.05010	-98.78	5.015
(iv)	10^{-155}	-10^{155}	10^{155}
(v)	6×10^{154}	5×10^{154}	-4×10^{154}