1. (10%) Let

\[
A = \begin{bmatrix}
1 & 3 \\
2 & 4 \\
0 & 0
\end{bmatrix}, \quad b = \begin{bmatrix}
5 \\
8 \\
3
\end{bmatrix}
\]

Find the linear least squares solution for \( Ax = b \).

2. (20%) Give the solution for each of the following Matlab codes.

(a) format short; fzero(@(t)(t^2-2.6*t+1.44), 1.76)

(b) fzero(@(x)(x^2-3.0*x+2.00), 0.36)

(c) roots([1, 6, 8])

(d) x=1.2; y=1.0; tol=0.001;
Nrun=20; num=1; delta=1.0;
while (++num < Nrun && delta > tol)
    y=x-(x^2-x+0.16)/(2*x-1);
    delta=abs(x-y);
    x=y;
end
x

(e) x=1.96; delta=1.0; tol=0.00001;
Nrun=30; num=0;
while (++num>Nrun && delta>tol)
    y=(x^2+2)/3;
    delta=abs(y-x);
    x=y;
end
x
3. (25%) Let
\[ A = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \]

(a) Find the eigenvalues and corresponding eigenvectors for matrix \( A \).
(b) Give the spectrum decomposition for \( A \).
(c) Find the singular values for \( A \).
(d) Find the singular value decomposition for \( A \).
(e) Give the Matlab commands to verify solutions for (b) and (c).

4. (25%) Let \( H \in \mathbb{R}^{n \times n} \) be a Householder matrix.
(a) Show that \( H \) is symmetric.
(b) Show that \( H \) is orthogonal.
(c) Show that \( \det(H) = -1 \).
(d) Find a Householder matrix \( H \) such that \( Hx = y \) for \( x, y \in \mathbb{R}^n \) with \( \|x\|_2 = \|y\|_2 \).

5. (10%) Let \( \lambda_1, \lambda_2, \ldots, \lambda_n \) be eigenvalues of \( A \in \mathbb{R}^{n \times n} \), show that \( \text{tr}(A^k) = \sum_{i=1}^{n} \lambda_i^k \).

6. (10%) Let \( A \in \mathbb{R}^{n \times n} \) is diagonalizable and that \( U^{-1}AU = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) \) with \( U = [v_1, v_2, \ldots, v_n] \) and \( |\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_n| \). Give an algorithm based on power method to find \( \lambda_1 \) and a corresponding eigenvector.