Test 2

1.(10%) Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix}$$

Find the linear least squares solution for $A\mathbf{x} = \mathbf{b}$.

2.(20%) Give the solution for each of the following Matlab codes.

(a) format short; fzero(@(t)(t*t-2.6*t+1.44), 1.76)

(b) fzero(@(x)(x*x-3.0*x+2.00), 0.36)

(c) roots([1, 6, 8])

(d)

```
 x=1.2;   y=1.0;  tol=0.001;
Nrun=20; num=1;  delta=1.0;
while (++num < Nrun && delta > tol)
  y=x-(x^2-x+0.16)/(2*x-1);
  delta=abs(x-y);
  x=y;
end
x
```

(e)

```
 x=1.96;   delta=1.0;  tol=0.00001;
Nrun=30;  num=0;
while (++num<Nrun && delta>tol)
  y=(x*x+2)/3;
  delta=abs(y-x);
  x=y;
end
x
```

3.(25%) Let

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) Find the eigenvalues and corresponding eigenvectors for matrix A.

(b) Give the spectrum decomposition for A.

(c) Find the singular values for A.

(d) Find the singular value decomposition for A.

(e) Give the Matlab commands to verify solutions for **(b)** and **(c)**.

4.(25%) Let $H \in R^{n\times n}$ be a Householder matrix.

(a) Show that H is symmetric.

(b) Show that H is orthogonal.

(c) Show that $det(H) = -1$.

(d) Find a Householder matrix H such that $H\mathbf{x} = \mathbf{y}$ for $\mathbf{x}, \mathbf{y} \in R^n$ with $\|\mathbf{x}\|_2 = \|\mathbf{y}\|_2$.

5.(10%) Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be eigenvalues of $A \in R^{n\times n}$, show that $tr(A^k) = \sum_{i=1}^{n} \lambda_i^k$.

6.(10%) Let $A \in R^{n\times n}$ is diagonalizable and that $U^{-1}AU = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$ with $U = [\mathbf{v}_1,\ \mathbf{v}_2,\ \cdots,\ \mathbf{v}_n]$ and $|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_n|$. Give an algorithm based on *power method* to find λ_1 and a corresponding eigenvector.