Test 1

Part I (60%)

1.(10%) Give a Taylor expansion for $cos(x)$ about $x = 0$.

2.(10%) Let the matrix B and the vector $\mathbf{x}$ be defined as follows.

$$B = \begin{bmatrix} 3 & 4 & -7 \\ 5 & -7 & 10 \\ -8 & 10 & -8 \end{bmatrix}, \quad \mathbf{x} = [1, -3, 1, 5]^t.$$

(a) Find $\|\mathbf{x}\|_{\mathbf{1}}$ and $\|\mathbf{x}\|_\infty$, respectively.

(b) Find $\|B\|_1$ and $\|B\|_\infty$, respectively.

3.(10%) Write down the matrix H obtained from the following Matlab code.

```
format rat
H=hilb(3)
```

4.(15%) Briefly describe what the following algorithms do.

(a) Doolittle Algorithm.

(b) Crout Algorithm.

(c) Cholesky Algorithm.

5.(15%) Give an example for each of the following requests.

(a) A tridiagonal matrix $A \in R^{3\times3}$.

(b) A symmetric and diagonally dominant matrix $B \in R^{3\times3}$.

(c) A Fibonacci number x_{10}, where $x_1 = 1$ and $x_2 = 1$.

Part II (40%)

6.(15%) Give the following Matlab code.

```
A=[1, 2, 3; 2, 6, 10; 3, 14, 28];
b=[1; 0; -8];
format short
X=A\b
```

(a) Find the *LU-decomposition* for A.

(b) What does the above Matlab code do?

(c) What is the output of X?

7.(15%) Suppose that a tridiagonal matrix $T \in R^{n \times n}$ is also diagonally dominant.

(a) Give an efficient algorithm to do $T = LU$.

(b) How many floating-point operations (flops) are needed for your algorithm?

8.(10%) Let $\|A\|_\infty = max_{\|\mathbf{x}\|_\infty = 1}\{\|A\mathbf{x}\|_\infty\}$ for $A = [a_{ij}] \in R^{m \times n}$ and $\mathbf{x} \in R^n$.

Prove that $\|A\|_\infty = max_{1 \le i \le m} \left[\sum_{j=1}^{n} |a_{ij}|\right]$