A Placement Test

Name: ______________ **StuID:**______________

1. Let $f(x) = e^{-x} - x$. Show that there exists an $x_0 \in [0, 1]$ such that $f(x_0) = 0$.

2. Define $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ for $x > 0$ and let $\gamma = \int_0^\infty e^{-x^2} dx$.

(a) Show that $\Gamma(x + 1) = x\Gamma(x),\ for\ x > 0$.

(b) Show that $\Gamma(\frac{1}{2}) = 2\gamma$ and find γ.

(c) Evaluate $\Gamma(\frac{3}{2})$.

3. Let $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$.

(a) Find the characteristic polynomial of matrix A.

(b) Find the eigenvalues and corresponding eigenvectors of matrix A.

(c) What are the singular values of A?

(d) Can the Cholesky algorithm be applied to decomposing A? Explain.

4. Evaluate the following integrals:

(a) $\int_0^{2\pi} sin(kx)sin(mx)dx$, where $k, m \geq 1,\ k \neq m$.

(b) $\int_0^{2\pi} cos(kx)cos(mx)dx$, where $k, m \geq 1,\ k \neq m$.

(c) $\int_0^{2\pi} sin(kx)cos(mx)dx$, where $k, m \geq 1$.

(d) $\int_0^{2\pi} sin(kx)sin(kx)dx$, where $k \geq 1$.

(e) $\int_0^{2\pi} cos(mx)cos(mx)dx$, where $m \geq 1$.