Introduction

- The central problems of Linear Algebra are to study the properties of matrices and to investigate the solutions of systems of linear equations
- Given A and b, find x for Ax=b; i.e., solve Ax=b
- Given A, find λ (lambda) such that Ax=λ x

Matrices and Linear Systems of Equations

- Matrix notations and operations
- Elementary row operations
- Row-echelon form
- Matrix inverse
- LU-Decomposition for Invertible Matrices
- LU-Decomposition with Partial Pivoting
- Some special matrices

Determinants

- Definition of Determinant det(A) or |A|
- Cofactor and minor at (i,j)-position of A
- Properties of determinants
- Examples and Applications

Vector Space and Linear Transform

- Vector Space, Subspace, Examples
- Null space, column space, row space of a matrix
- Spanning sets, Linear Independence, Basis, Dimension
- Rank, Nullity, and the Fundamental Matrix Spaces
- Matrix Transformation from R^n to R^m
- Kernel and Image of a Linear Transform

    Projection, rotation, scaling

    Gauss transform

    Householder transform (elementary reflector)

    Jacobi transform (Givens' rotation)

- An affine transform is in general not a linear transform

Orthogonality

- Motivation – More intuitive
- Inner product and projection
- Orthogonal vectors and linear independence
- Orthogonal complement
- Projection and least squares approximation
- Orthonormal bases and orthogonal matrices
- Gram-Schmidt orthogonalization process
- QR factorization

Eigenvalues and Eigenvectors

- Definitions and Examples
- Properties of eigenvalues and eigenvectors
- Diagonalization of matrices
- Similarity transformation and triangularization
  Positive definitive (Semi-definite) matrices
- Spectrum decomposition
- Singular Value Decomposition (SVD)
- A Markov process
- Differential equations and exp(A)
- Algebraic multiplicity and geometric multiplicity
- Minimal polynomials and Jordan blocks