Linear Algebra

Let $A \in R^{n \times n}$ and $\mathbf{x}, \mathbf{b} \in R^n$ with $n = 4$ be defined as follows.

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 4 & 1 & -1 & 4 \\ 8 & -10 & 3 & 2 \\ 2 & 2 & -2 & 5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 6 \\ 2 \end{bmatrix}$$

A linear system of equations $A\mathbf{x} = \mathbf{b}$ can be written as

$$\begin{array}{rcrcrcrcl} 2w & - & x & & & + & z & = & 1 \\ 4w & + & x & - & y & + & 4z & = & 0 \\ 8w & - & 10x & + & 3y & + & 2z & = & 6 \\ 2w & + & 2x & - & 2y & + & 5z & = & 2 \end{array}, \quad \left[\begin{array}{cccc|c} 2 & -1 & 0 & 1 & 1 \\ 4 & 1 & -1 & 4 & 0 \\ 8 & -10 & 3 & 2 & 6 \\ 2 & 2 & -2 & 5 & 2 \end{array}\right]$$

Use Matlab commands to answer the following questions.

(a) Input matrix A.

(b) Input matrix b.

(c) Apply Gaussian elimination with partial pivoting to solve $A\mathbf{x} = \mathbf{b}$.

(d) In **(c)**, report the matrices of P, L, U, respectively, where P is a permutation metrix, L is unit $lower - \Delta$ and U is $upper - \Delta$.

(e) Find the determinant of A.

(f) Find the rank of A.

(g) Find the inverse matrix of A in a rational form.

(h) Find the characteristic polynomial $p(x)$ of A.

(i) Find the roots of $p(x) = 0$.

(j) Find the eigenvalues of A.

(k) Find the QR factorization of A.

(l) Find the singular value decomposition of A.

(m) Find $\|A\|_1$, $\|A\|_2$, and $\|A\|_\infty$, respectively.

(n) Find $LU-decomposition$ for matrix A, where L is unit $lower-\Delta$ and U is $upper-\Delta$.